Math 157 Fall 2013 — Homework 8

Problem 1. Let $s : [1, 7] \to \mathbb{R}$ and $t : [1, 7] \to \mathbb{R}$ be step functions given by

$$s(x) = \begin{cases} 3 & \text{if } 1 < x < 3 \\ 5 & \text{if } 3 < x < 5 \\ 4 & \text{if } 5 < x < 7 \end{cases} \qquad t(x) = \begin{cases} 2 & \text{if } 1 < x < 2 \\ 1 & \text{if } 2 < x < 4 \\ 3 & \text{if } 4 < x < 6 \\ 2 & \text{if } 6 < x < 7 \end{cases}$$

(a) Sketch the graphs of s and t.

(b) Describe $s + t$ and sketch its graph.

(c) Find a partition of $[1, 7]$ for which s, t, and $s + t$ are constant on its open subintervals.

(d) Compute the integrals $\int_1^7 s$, and $\int_1^7 t$, $\int_1^7 s + t$.

(e) You should have found that $\int_1^7 s + t = \int_1^7 s + \int_1^7 t$. Give a step by step arithmetic argument to show explicitly that

$$\int_1^7 s + t = \int_1^7 s + \int_1^7 t.$$

Problem 2. Read sections 1.16-1.17 in Apostol.

Problem 3. Let $f : [0, 1] \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 2 & \text{if } x \in [0, 1] \setminus \mathbb{Q} \end{cases}$$

Compute $\overline{I}(f)$ and $\underline{I}(f)$, the upper and lower integrals of f over the interval $[0, 1]$.

Problem 4. Read sections 1.18-1.27 in Apostol.

Problem 5. Let $f(x) = x^2$ and $g(x) = 2x + 1$.

(a) Sketch a picture of the graphs of f, g, and $f + g$ over the interval $[0, 2]$.

(b) Compute $\int_0^2 f$, $\int_0^2 g$, and $\int_0^2 f + g$.