# Math 157 Fall 2013 — Problem Set 2: Due Nov 26

**Problem 1. [2 points]** Use the four fundamental properties of sine and cosine on page 95 of the Apostol's book to prove that

$$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}.$$

Then, use the other properties of sine and cosine listed in sections 2.5, 2.6, and 2.7 to compute

$$\int_0^\pi \left|\frac{1}{2} + \cos(t)\right| dt$$

**Problem 2. [1 point each]** True or False. Completely justify your answers.

(a) $\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{3}}{2\sqrt{2}}$.

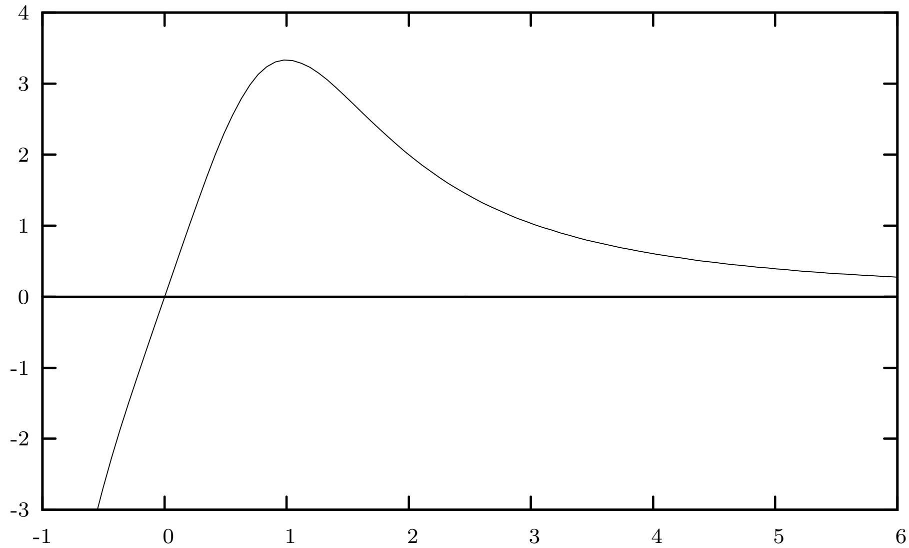
(b) If $f$ is increasing, then $f(b)(b-a) \geq \int_a^b f$.

(c) If $f$ is integrable and satisfies $f(t+1) = f(t)$ for all $t$, then $A(x) = A(x+1)$ where $A$ is defined by $A(x) = \int_a^x f(t)$.

(d) If $f$ is increasing, then the function $A$ defined by $A(x) = \int_a^x f(t)$ is also increasing.

**Problem 3. [2 points]** Let $f(t) = t - [t] + \frac{1}{2}$ and $A(x) = \int_0^x f(t)dt$. Sketch the graph of $A$ on the interval $[-10, 10]$.

**Problem 4.** [**3 points**] Let $A(x) = \int_1^x \frac{10t\,dt}{2+t^3}$ for $x \geq -\sqrt[3]{2}$. Here's a sketch of $y = \frac{10t}{2+t^3}$.




(a) Determine $A(x)$ for a few values of $x$, say $x = -1, 0, 1, 5$, Just eyeball it, or use a couple of rectangles to approximate.

(b) $A$ has a minimum on $(-\sqrt[3]{2}, \infty)$. What is it?

(c) Sketch the graph of $A$.

**Problem 5.** [**Bonus. 2 points**] Find a function $f$ so that

$$\int_1^x f(t)dt = x^2 + 2x + 5$$

or prove that no such function exists.