Problem 1. Carefully state:

(a) The mean value theorem.

(b) L'Hôpital's Rule.

(c) The fundamental theorem of calculus.

Problem 2. Matching

- $\displaystyle\int_1^\infty \frac{1}{t^4}dt$
- $\displaystyle\int_1^\infty \frac{1}{1+t^4}dt$
- $\displaystyle\int_1^\infty \frac{t}{1+t^4}dt$

- $\frac{1}{3}$
- $\frac{\pi}{8}$
- $\frac{\operatorname{arccoth}(\sqrt{2})}{4\sqrt{2}}$

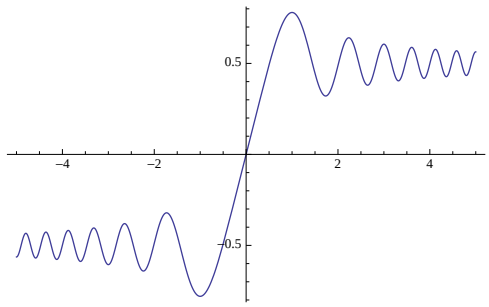
- $\displaystyle\int_0^\infty \frac{t}{e^t}dt$
- $\displaystyle\int_0^\infty \frac{t}{e^{t^2}}dt$
- $\displaystyle\int_0^\infty \frac{1}{e^{t^2}}dt$

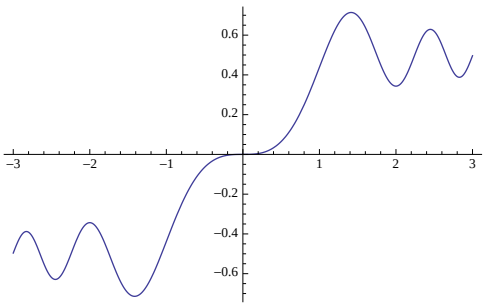
- $\frac{1}{2}$
- 1
- $\frac{\sqrt{\pi}}{2}$.

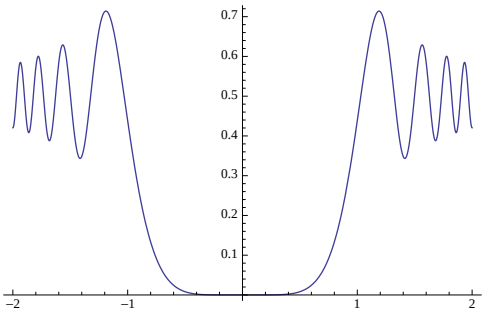
Problem 3. The so called *Fresnel integral sine function* is defined to be

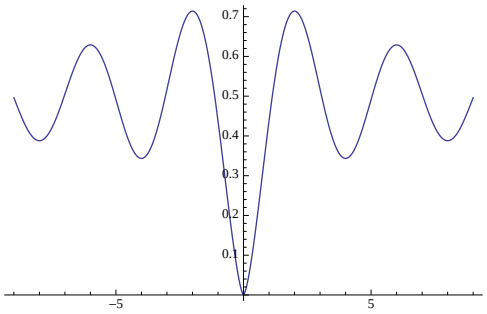
$$S(x) = \int_0^x \sin\left(\frac{\pi t^2}{2}\right)\, dt.$$

Which is the graph of S?

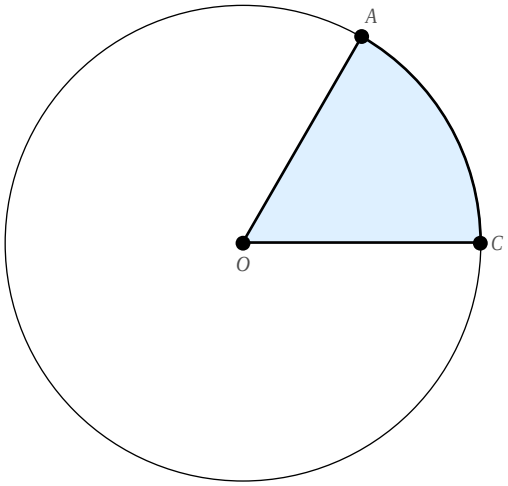




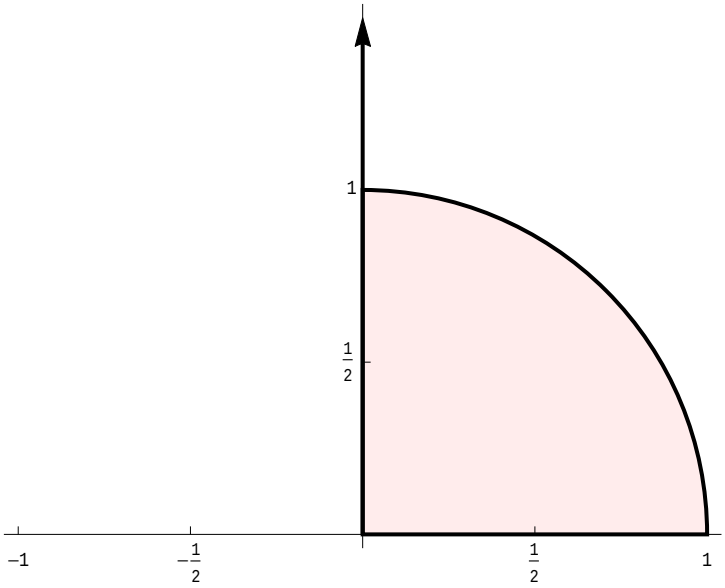




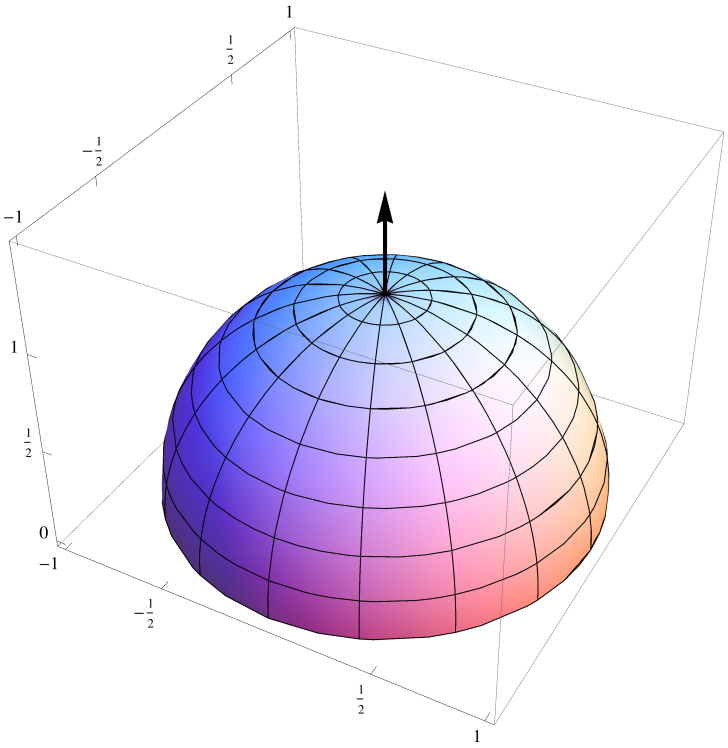
Problem 4. Here is the unit circle. The shaded sector AOC has area $\frac{t}{2}$. Use the integral formula for arclength to compute the length of the circular arc from A to C.



Problem 5.
Consider the region in the first quadrant bound by the unit circle.



If we rotate this region around the y-axis we obtain the upper half of the unit hemisphere.



Your problem: Find the volume of the northern hemisphere of the unit sphere in two different ways:

Problem 5. Continued.

(a) By discs.

(b) By shells.

EXAM

Midterm Exam

Math 158: Spring 2013

Tuesday, March 17

- Make sure your solutions are clearly and carefully written. Proofread.
- Show your work, but not your scratchwork. Neatness counts.
- Each part of each problem is worth one points for a total of nine possible points.

Success!