Math 158 Spring 2014 **Homework 2**

Some new problems

Problem 1. Read through section 4.20 in the text and do exercises 1, 8, 9 in section 4.19 (page 191) and exercises 2, 3, 19, 21, 28 in section 4.21 (page 194-196)

Problem 2. True or False: Let $f(x) = (x-1)^{\frac{2}{3}}$. Since $f(0) = 1$ and $f(2) = 1$, there is some point $c \in (0, 2)$ with $f'(c) = 0$.

Problem 3. Prove:

(a) If f satisfies $|f(s) - f(t)| \leq |s - t|$ for all s, t, then f is continuous.

(b) If f satisfies $|f(s) - f(t)| \leq |s - t|^2$ for all s, t, then f is constant.

Problem 4. It is hard, or maybe even impossible, to determine

$$\int_{-\frac{\sqrt{\pi}}{2}}^{-\frac{\sqrt{\pi}}{2}} \cos(x^2)dx$$

exactly. But $g(x) = \cos(x^2)$ can be approximated by a polynomial. Your problem: find a polynomial $p(x) = ax^4 + bx^3 + cx^2 + dx + e$ that satisfies $p(0) = g(0)$, $p'(0) = g'(0)$, $p''(0) = g''(0)$, $p'''(0) = g'''(0)$, and $p''''(0) = g''''(0)$. Use a computer to graph g and p in the same picture. Approximate $\int_{-\frac{\sqrt{\pi}}{2}}^{-\frac{\sqrt{\pi}}{2}} \cos(x^2)dx$ by computing

$$\int_{-\frac{\sqrt{\pi}}{2}}^{-\frac{\sqrt{\pi}}{2}} p(x)dx$$

instead.

Problem 5. Suppose that $f(4) = 0$, $g(4) = 0$, $f'(4) = 7$, and $g'(4) = -1$. Prove that $\lim_{x \to 4} \frac{f(x)}{g(x)}$ exists and compute it.

Some review problems

Problem 6. Let f be a function defined on an open neighborhood of c. Define the statement "f is differentiable at c" and the number $f'(c)$.

Problem 7. Prove that if f is differentiable at $x = c$ then f is continuous at $x = c$.

Problem 8. Prove that if $f : \mathbb{R} \to \mathbb{R}$ is differentiable and $f(c) \geq f(x)$ for all $x \in \mathbb{R}$, then $f'(c) = 0$.

Problem 9. Prove that if $f'(x) > 0$ for all $x \in (0, 1)$, then f is increasing on $(0, 1)$.