Math 157 Fall 2013 — Problem Set 3: Due Dec 3

Problem 1. **[1 point each]** Compute:

(a) $\lim_{x\to 2} \dfrac{3x^2 - 5x - 2}{2x^2 - 9x + 10}$.

(b) $\lim_{x\to 0} x^2 \sin\left(\dfrac{1}{x}\right)$.

(c) $\lim_{x\to 0} \dfrac{\sqrt{1-x} - 1}{x}$.

(d) $\lim_{x\to 0} \dfrac{\tan(2x)}{\sin 3x}$.

Problem 2. **[1 point each]** True or False. Give proofs or counterexamples.

(a) If $\lim_{x\to a} f(x)$ does not exist and $\lim_{x\to a} g(x) = L$, then $\lim_{x\to a} f(x) + g(x)$ does not exist.

(b) If $\lim_{x\to a} f(x)$ does not exist and $\lim_{x\to a} g(x)$ does not exist, then $\lim_{x\to a} f(x)g(x)$ does not exist.

(c) If $\lim_{x\to a} f(x)$ does not exist and $\lim_{x\to a} g(x) = L$, then $\lim_{x\to a} f(x)g(x)$ does not exist.

(d) If $f(x) \geq 0$ for all x in an interval $[a, b]$ and $\int_a^b f = 0$, then $f = 0$.

Problem 3. **[1 point each]** Definitions and theorems

(a) Let f be a function defined on an open neighborhood of c. Define the statement "f is continuous at c."

(b) State Bolzano's theorem.

(c) State the intermediate value theorem.

(d) State the mean value theorem for integrals.

Problem 4. **[Bonus 2 points]** Prove:

Theorem. *If f is continuous, $f(x) \geq 0$ for all $x \in [a, b]$, and $\int_a^b f = 0$ then $f(x) = 0$ for all $x \in [a, b]$.*