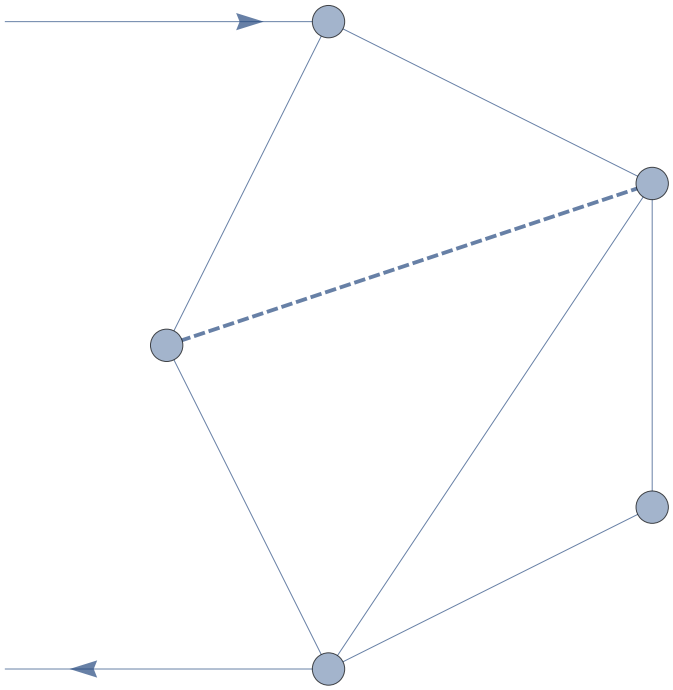
Math 634 Spring 2016 — Final Exam Problem 1

Problem 1. Consider the graph below as an electrical circuit where each edge has resistance 1 and a current of size 1 enters the graph at the source at the top and exits the graph at the sink at the bottom.



(a) Compute the number of spanning trees. Do it several ways: multiply the nonzero eigenvalues of the Laplacian, compute the determinant of the reduced Laplacian, draw of picture of all of them.

(b) Determine the current flowing in each edge by using Kirchoff's laws.

(c) Determine the current flowing in the dotted edge by Theorem 1 in Section II.1 in the book.

(d) Compute the page ranks for the directed graph obtained by connecting the sink to the source and considering the edges to be directed according to the flow of the current.