Problem 1. Prove that $\mathrm{rad}(G) \leq \mathrm{diam}(G) \leq 2\,\mathrm{rad}(G)$ for any graph G.

Problem 2. A little experimenting will tell you that K_3 is the disjoint union paths of length 1 and 2; K_4 is the disjoint union of paths of length $1, 2$, and 3; K_5 is the disjoint union of paths of length $1, 2, 3, 4$. Your problem: prove that K_n is the disjoint union of paths of lengths $1, 2, \ldots, n-1$.

Problem 3. Prove that every spatial embedding of $K_{4,4}$ has a pair of linked rectangles.

Problem 4. Prove or disprove: Every embedding of $G = (K_{3,3} - e) + K_1$ has a pair of linked cycles.

Problem 5. Give an example of graphs G and H so that H is minor of G but not a subgraph of a subdivision of G.

Problem 6. Draw all the graphs that can be obtained from K_6 by $\Delta - Y$ exchanges.

Problem 7. Are K_5 and $K_{3,3}$ related by $\Delta - Y$ exchanges?

Problem 8. Find a list of forbidden minors to characterize forests.

Problem 9. The *crossing number* of a graph is the smallest number of crossings that occur in a diagram of a spatial embedding of the graph. Prove that if G has crossing number ≤ 1 then G can be embedded on a torus, but not conversely.

Problem 10. Prove that K_n can be embedded on the torus if and only if $n \leq 7$.

Problem 11. Suppose G is a maximal planar graph. That is, a planar graph such that adding any edge to G results in a nonplanar graph. Prove that every face of G is a triangle.