Math 1220 – Practice Final

1. Evaluate these integrals:
   (a) $\int ye^{-2y}dy$
   (b) $\int \frac{x-2}{(x+1)(x-1)}dx$
   (c) $\int_0^2 \frac{1}{(x-1)^2}dx$ (Hint: Notice that $\frac{1}{(x-1)^2}$ blows up at $x = 1$!!)
   (d) $\int xsin(x)dx$
   (e) $\int \frac{x^2}{x^3+1}dx$
   (f) $\int \frac{x+1}{x(x-1)}dx$

2. (a) Find the first four non-zero terms of $f(x) = sin(x^2)$ in its MacLaurin series.
   (b) Use your answer from (a) to estimate $\int_0^{1/2} sin(x^2)dx$.

3. Find the convergence set for
$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n!}$$

4. Find the convergence set for
$$\sum_{n=1}^{\infty} \frac{(x-4)^n}{2^n}$$

5. Find a power series for $F(x) = \int x^4 e^{x^2}dx$ and state its radius of convergence.

6. Evaluate these integrals:
   (a) $\int cos^3(x)dx$
   (b) $\int_0^1 \frac{4y}{\sqrt{y^2+6}}dy$.

7. Find the limit, if it exists.
   (a) $\lim_{x\to 0} \frac{sin(5x)}{tan(4x)}$
   (b) $\lim_{x\to\infty} (2x)^{1/3x}$
   (c) $\lim_{x\to 0} \frac{xcos(\pi x)}{2x-1}$
   (d) $\lim_{x\to\infty} (2x)^{1/x}$

8. Determine if each series is absolutely convergent, conditionally convergent, or divergent.
   (a) $\sum_{n=1}^{\infty} \frac{3n-10}{2n+3}$

(b) $\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$ (Hint: Recall that $\lim_{n\to\infty} (1+1/n)^n = e$.)

(c) $\sum_{n=1}^{\infty} \frac{\sqrt{3n}}{n^3+5}$

(d) $\sum_{n=1}^{\infty} \frac{n-2}{4n+1}$

(e) $\sum_{n=1}^{\infty} \frac{n^4(-3)^n}{(n+2)!}$

(f) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2 3^{n+1}}{e^{2n}}$

9. Find a power series that represents $f(x) = \frac{1}{1-2x} + e^{-3x}$. (Write out the terms through x^3.) State its radius of convergence.

10. Find a power series that represents $f(x) = \frac{3}{1+x} + cos(x)$. (Write out the terms through x^4.) State its radius of convergence.

11. For $f(x) = \frac{3}{2+x}$,

    (a) Find the Taylor polynomial of order 3 centered about $a = 1$.

    (b) Approximate $f(1.3)$ using the Taylor polynomial in part (a).

    (c) Find a bound for the error in your approximation.

12. For $f(x) = \frac{1}{3+x}$,

    (a) Find the Taylor polynomial of order 3 centered about $a = 2$.

    (b) Approximate $f(2.3)$ using the Taylor polynomial in part (a).

    (c) Find a bound for the error in your approximation.