

Math 143 – Practice Midterm Exam 2

1. Use the geometric series to find the Taylor series for $f(x) = \ln(1+x)$ around $x = 0$. Determine the radius of convergence of this series. Do the same for $f(x) = 1/(1+x)^2$.
2. Use the geometric series to find the Taylor series for $f(x) = 1/(4+x^2)$ around $x = 0$. Determine the radius of convergence for this series.
3. Find the convergence set for the following power series. Also analyze the type of convergence (or divergence) at the endpoints of the convergence set.

(a)
$$\sum_{n=1}^{\infty} \frac{(3x+1)^n}{n2^n}.$$

(b)
$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n!}.$$

4. Find the following limits:

(a)
$$\lim_{n \rightarrow \infty} e^{-n} \sin(n)$$

(b)
$$\lim_{n \rightarrow \infty} (2n)^{1/2n}$$

5. Determine whether the following infinite series converge or diverge. If a series converge, determine whether the convergence is absolute or conditional.

(a)
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\tan^{-1} n}{1+n}.$$

(b)
$$\sum_{n=1}^{\infty} \frac{n^{100}}{n!}$$

(c)
$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^\pi}.$$

(d)
$$\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}.$$

6. Find the Taylor series for $f(x) = \sqrt{x-2}$ around the point $x = 3$. Find the radius of convergence of this series.
7. Find a power series that represents $f(x) = \frac{3}{1+x} + \cos(x)$. (Write out the terms through x^4 .) State its radius of convergence.
8. For $f(x) = \frac{3}{2+x}$,
 - (a) Find the Taylor polynomial of order 3 centered about $a = 1$.
 - (b) Approximate $f(1.3)$ using the Taylor polynomial in part (a).
 - (c) Find a bound for the error in your approximation.