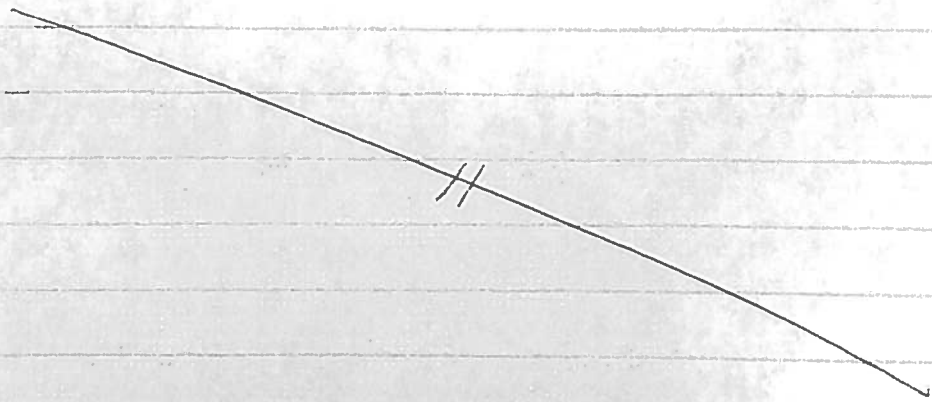


Tashu (2/8/2013)

Rigid inner forms

motivation

F real / p-adic \mathbb{R} / \mathbb{Q}_p G_T : conn. red / F $\hookrightarrow G_T = \widehat{G}_T \rtimes W_F$
 $\Gamma = \text{Gal}(\overline{F}/F)$ $W_F = \text{abs. Weil gp.}$ $W'_F = L_F = \begin{cases} W_F & F = \mathbb{R} \\ W_F \text{ISO}(2) & F \text{ p-adic} \end{cases}$



$\varphi: L_F \rightarrow L_G$ L-parameter.
 $\downarrow \# \downarrow$
 W_F

Assume φ tempered.
 $\Rightarrow \Pi_G^G$ a finite set of mod char depend on φ of $G(F)$.
 Expected.

Fix a Whittaker datum

$w = (B, \psi)$
 $\uparrow \quad \uparrow$
 Borel generic $Z(F) \rightarrow G^X$.

$S_\varphi = \text{Cent}(\text{Im } \varphi, \hat{G})$. Expected: $\Pi_G^G \xleftrightarrow{(1-1)} \text{Int}(\pi_0(S_\varphi / Z(\hat{G})^\Gamma)$
 $\pi \longmapsto \langle \pi, - \rangle_w$

Conditions

- ① $\langle \pi, - \rangle_w = 1$
 $\Leftrightarrow \pi$ is ω -generic
- ② $\sum_{\pi \in \Pi_G^G} \langle \pi, 1 \rangle_w \Theta_\pi$ is stable.
- ③ If $L_H \rightarrow L_G$ is an endoscopic L-embedding and φ factors thru it

$\sum_{\mathfrak{H}} \Delta(\omega)(\mathfrak{H}, \mathfrak{H}) \sum_{\pi \in \Pi_G^H} \langle \pi, 1 \rangle \Theta_\pi(\mathfrak{H}^H) = \sum_{\pi \in \Pi_G^G} \langle \pi, s \rangle_\omega \Theta_\pi(\mathfrak{H})$
 (H, s)

called Langlands-Shelstad transfer factor.

Now we consider "non q-split" G'_F !

\exists a unique G q-split $\Sigma: G \rightarrow G'_F$, for all $\sigma \in \Gamma$, $\Sigma^{-1}(\sigma(\Sigma)) \in \text{Int}(G)_F$.
 $L_{G'_F} \cong L_G$

- problem
- No natural normalization of the transfer factor \Rightarrow No parametrization of $\Pi_{G'_F}$
 - No Whittaker datum.
 - Not invariant under act of endoscopic datum
- any normalization is

Arthur's suggestions:

- \exists of add. fit. ? - spectral transfer factor $\Delta(\varphi, \pi)$.
- mediating fit. $\rho(\sigma, \Sigma)$.

over \mathbb{R} , Shelstad proved this.

Vojta's: Define Π_g to consist of iso-classes of tuples (G', ξ, π) and seek a parameterization.

Problem Not possible.

too many auto. $(\mathbb{Q}_2/\mathbb{K}, \text{id}, D_F^+) \cong (\mathbb{Q}_2/\mathbb{R}, \text{id}, P_C^-)$.

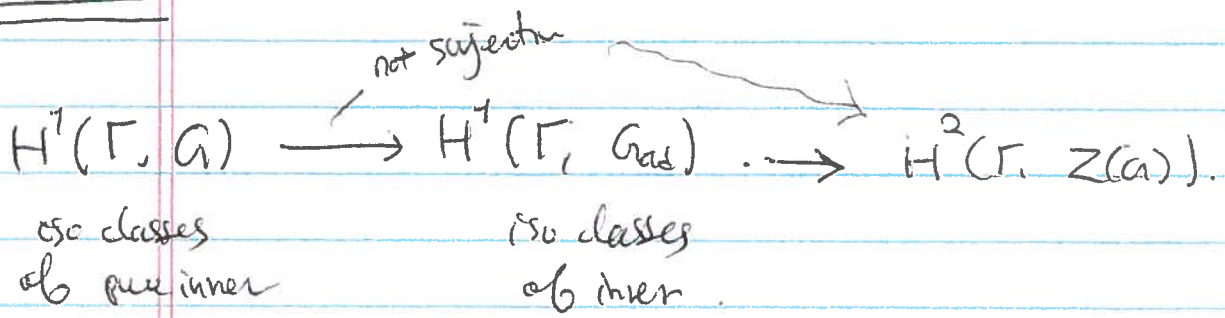
Solution Enrich each inner twist

(G', ξ) by $z \in Z^1(\Gamma, G)$ with

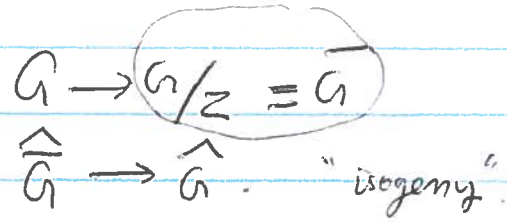
$$\xi^T \circ (\xi) = \text{Ad}(z(\sigma)).$$

- possible to define $\Delta[\text{Im}, z]$ & $\leftarrow, \rightarrow_{\text{Im}, z}$.

Problem Not every (G', ξ) admits a z_e

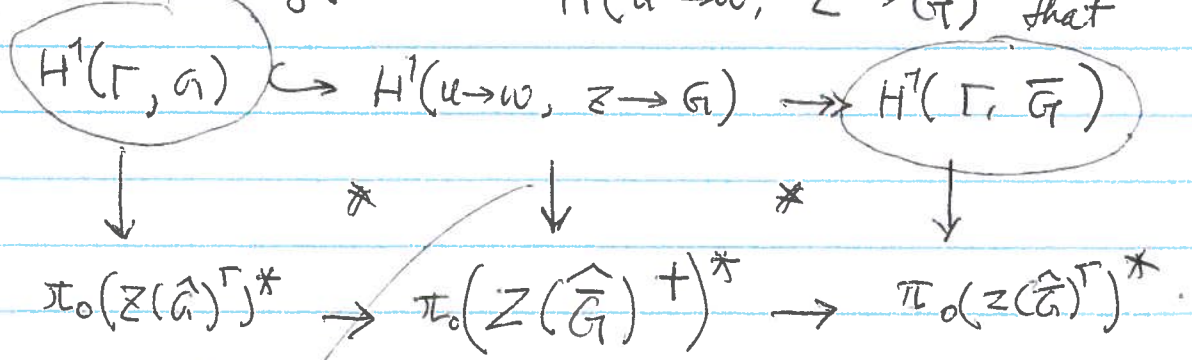


Let $Z \in G$. "finite central subgroup".



one can define a cohomology class $H^1(u \rightarrow w, Z \rightarrow G)$ that

fits into



bijective if $\begin{cases} F: \text{pairs} \\ G: \text{tors} \end{cases}$

What is known: • IR: 1) diagram + stability + end transfer works for all tempered φ .

(7/8)

2) rigid over twists \longleftrightarrow strong real forms

• p-adic:

$$\sum_{\varphi \in H} \Delta[\mu, z](\varphi^H, \tau) \sum_{\pi \in \pi_G^H} \langle \pi, \tau \rangle \Theta_{\pi}(\varphi^H) = \sum_{\pi \in \pi_G^{G'}} \langle \omega(\xi, z, \pi), \dot{s} \rangle \Theta_{\pi}(\varphi^H)$$

$\begin{matrix} H? \\ 1 \end{matrix}$ if $H=G$.

$\begin{matrix} \text{if } G=H \\ \text{split form} \end{matrix}$