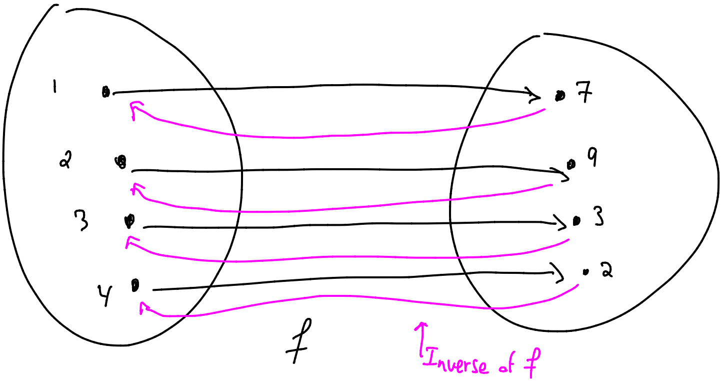
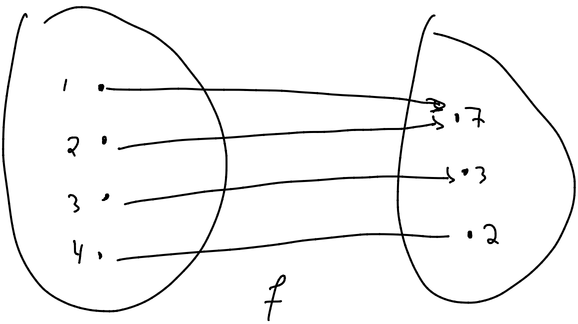
Inverse Functions (§5.1



What should the inverse of f do?



Can f have an inverse?

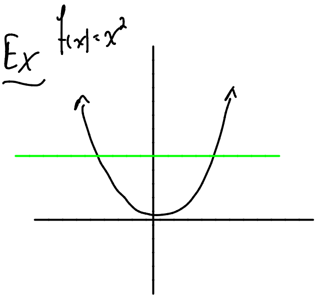
No: Where would 7 go?

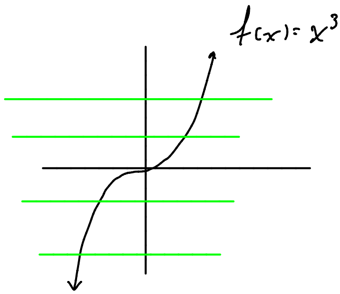
Def A functione is one-to-one if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$

(f never takes the same value twice)

Horizontal Line Test A function f is one-to-one if and only if every horizontal line intersects the graph of f at most once.

Ex





Fails horizontal line test

$\Downarrow$

Not one-to-one

Passes horizontal line test

$\Downarrow$

One-to-one

Def Let f be a one-to-one function w/ domain A and range B, then its inverse function f^{-1} has domain B and range A and is defined by

$$f^{-1}(y) = x \iff f(x) = y$$

for any y in B.

- domain of f^{-1} = range of f
- range of f^{-1} = domain of f
- $f^{-1}(f(x)) = x$ and $f(f^{-1}(y)) = y$

Ex. If $f(x) = x^3$, then $f^{-1}(x) = x^{1/3}$.

Check
- $f^{-1}(f(x)) = f^{-1}(x^3) = (x^3)^{1/3} = x$ ✓
- $f(f^{-1}(y)) = f(y^{1/3}) = (y^{1/3})^3 = y$ ✓

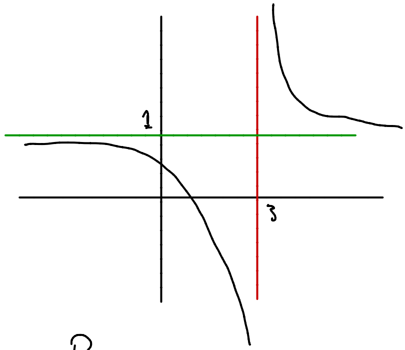
A Strategy for finding inverses:

1) Write $y = f(x)$
2) Solve this equation for x in terms of y (if possible)
3) To express f^{-1} as a function of x, interchange x and y.

Ex. $f(x) = x^5 - 7$

1) $y = x^5 - 7$
2) $y + 7 = x^5$
   $x = (y+7)^{1/5}$
3) $f^{-1}(x) = (x+7)^{1/5}$

Ex $f(x) = \dfrac{x+2}{x-3}$



Passes horizontal line test

$$\lim_{x\to\infty} f(x) = 1$$

$$\lim_{x\to-\infty} f(x) = 1$$

$$\lim_{x\to 3^-} f(x) = -\infty$$

$$\lim_{x\to 3^+} f(x) = \infty$$

1) $y = \dfrac{x+2}{x-3}$

2) $(x-3)y = x+2$

$$yx - 3y = x+2$$

$$yx - x = 3y+2$$

$$(y-1)x = 3y+2$$

$$x = \frac{3y+2}{y-1}$$

3) $f^{-1}(x) = \dfrac{3x+2}{x-1}$

Let's Check our solution:

$$f^{-1}(f(x)) = f^{-1}\left(\frac{x+2}{x-3}\right)$$

$$= \frac{3\left(\frac{x+2}{x-3}\right)+2}{\left(\frac{x+2}{x-3}\right)-1}$$

$$= \frac{3(x+2)+2(x-3)}{(x+2)-(x-3)}$$

$$= \frac{3x+6+2x-6}{x+2-x+3}$$

$$= \frac{5x}{5}$$

$$= x$$

A similar process shows that $f(f^{-1}(x)) = x$.