General Logarithmic and Exponential Functions

For any positive number a, $\quad a = e^{\ln(a)}$

So $\quad a^x = \left(e^{\ln(a)}\right)^x = e^{x\ln(a)}$.

So, $\quad \dfrac{d}{dx} a^x = \dfrac{d}{dx} e^{x\ln(a)} = e^{x\ln(a)} \cdot \ln(a) = a^x \ln(a)$

$$\boxed{\frac{d}{dx} a^x = a^x \cdot \ln(a)}$$

Using this we can conclude:

$$\boxed{\int a^x \, dx = \frac{a^x}{\ln(a)} + C}$$

Ex $\quad \displaystyle\int x \pi^{x^2} dx = \frac{1}{2}\int \pi^u \, du = \frac{1}{2\ln(\pi)} \pi^u + C$

$u = x^2$

$\dfrac{du}{dx} = 2x$

$$= \frac{\pi^{x^2}}{2\ln(\pi)}$$

General logs

For any positive number a, we define $\log_a x$ to be the inverse of a^x.

This means that $a^{\log_a x} = x$.

By taking ln of both sides we obtain:

$$\ln\left(a^{\log_a x}\right) = \ln x$$

$$\Rightarrow \log_a x \cdot \ln(a) = \ln(x)$$

$$\Rightarrow \log_a x = \frac{\ln(x)}{\ln(a)}$$

$$\boxed{\frac{d}{dx}\log_a x = \frac{1}{x\ln(a)}}$$