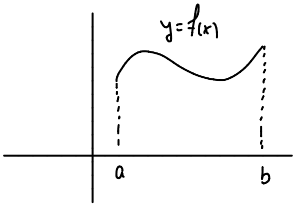
Last time we saw $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$.

Ex $\int \frac{x+1}{1+x^2}\,dx = \int \frac{x}{1+x^2}\,dx + \int \frac{1}{1+x^2}\,dx$

$= \frac{1}{2}\ln|1+x^2| + \arctan x + C$

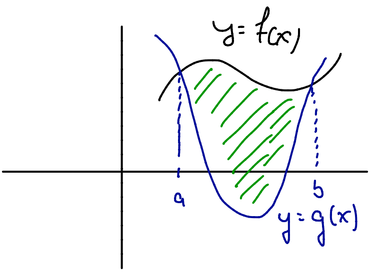
Area Between Curves (§7.1)

Recall:



$\int_a^b f(x)\,dx$ = Net area between the curve $y=f(x)$ and the x-axis

Given two functions $f(x)$ and $g(x)$:



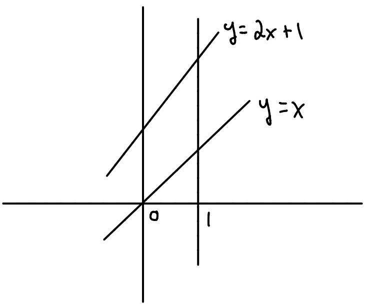
How do you calculate the (green) area between the curves $y=f(x)$ and $y=g(x)$?

A: $\int_a^b [f(x)-g(x)]\,dx$

In general, the area A of a region bound by the curves $y=f(x)$, $y=g(x)$, and the lines $x=a$, $x=b$, where f and g are continuous and $f(x) \geq g(x)$ for all x in $[a,b]$ is

$$A = \int_a^b \left[f(x) - g(x)\right]$$

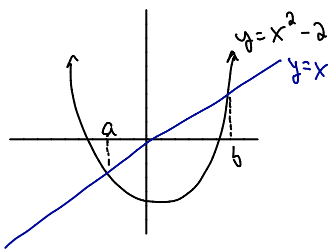
Ex Find the area of the trapezoid bounded by the lines $y = x$, $y = 2x+1$, $x=0$, and $x=1$.



$$A = \int_0^1 \Big[\underbrace{(2x+1)}_{\text{Top function}} - \underbrace{x}_{\text{Bottom function}}\Big]\, dx$$

$$= \int_0^1 (x+1)\, dx = \left(\frac{x^2}{2} + x\right)\Big|_0^1 = \frac{3}{2}$$

Ex Find the area bounded by the curves $y = x^2 - 2$ and $y = x$



Need to find a and b.

$x^2 - 2 = x$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = -1, 2$

So, $a = -1$ and $b = 2$.

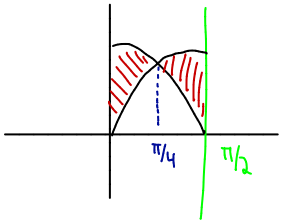
$$A = \int_{-1}^{2} \left(x - (x^2 - 2)\right) dx$$

$$= \left(\frac{x^2}{2} - \frac{x^3}{3} + 2x\right)\Big|_{-1}^{2}$$

$$= \frac{9}{2}$$

Ex Find the area of the region bounded by the curves

$$y = \sin x, \quad y = \cos x, \quad x = 0, \quad x = \frac{\pi}{2}$$



The thing to note here is that the function that is on top switches. It switches at the point where $\sin x = \cos x$. Between 0 and $\pi/2$, this occurs at $x = \pi/4$.

$$A = \int_0^{\pi/4} (\overset{y_{top}}{\cos x} - \overset{y_{bottom}}{\sin x})\,dx + \int_{\pi/4}^{\pi/2} (\overset{y_{top}}{\sin x} - \overset{y_{bottom}}{\cos x})\,dx$$

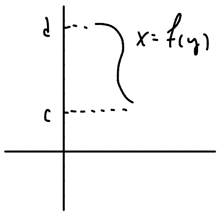
$$= (\sin x + \cos x)\Big|_0^{\pi/4} + (-\cos x - \sin x)\Big|_{\pi/4}^{\pi/2}$$

$$= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1\right) + \left(-1 - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)\right)$$

$$= 2\sqrt{2} - 2$$

$$= 2(\sqrt{2} - 1)$$

Integrating with respect to y:



$\int_c^d f(y)\,dy$ = Net area between the curve $x = f(y)$ and the y-axis from $y = c$ to $y = d$.