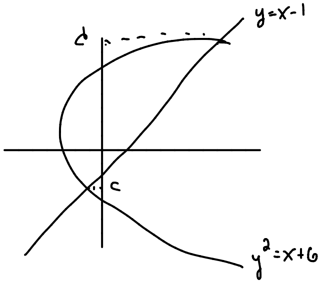
Ex Find the area of the region enclosed by the line $y=x-1$ and the parabola $y^2=2x+6$



Here, it is easier to integrate wrt y:

$$A=\int_c^d (X_R-X_L)\,dy$$

Find c and d:

$X=y+1$ and $x=\frac{1}{2}y^2-3$

Set $y+1=\frac{1}{2}y^2-3$

$\Rightarrow \frac{1}{2}y^2-y-4=0$

$\Rightarrow y^2-2y-8=0$

$\Rightarrow (y-4)(y+2)=0 \Rightarrow c=-2,\ d=4$

$$A=\int_{-2}^{4}\left(y+1-\left(\tfrac{1}{2}y^2-3\right)\right)dy$$

$$=\int_{-2}^{4}\left(-\tfrac{1}{2}y^2+y+4\right)dy$$

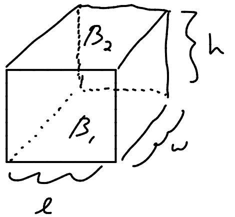
$$=\left(-\tfrac{1}{6}y^3+\tfrac{y^2}{2}+4y\right)\Big|_{-2}^{4}$$

$$=18$$

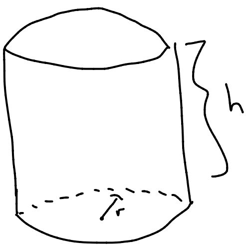
Volumes (§7.2)

Def A <u>cylinder</u> is a solid object bounded by a plane region B_1, called the <u>base</u>, and by a parallel and congruent region B_2.

Ex

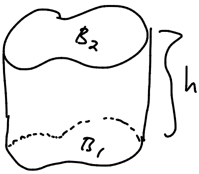


$V = l \cdot w \cdot h = \text{Area}(B_1) \cdot h$

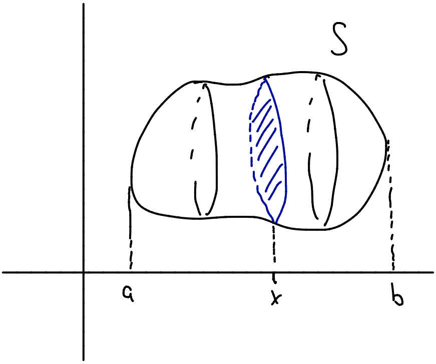


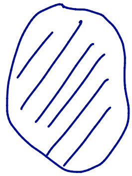
$V = \pi r^2 \cdot h = \text{Area}(B_1) \cdot h$

In general, the volume of a cylinder w/ base B and height h is $V = \text{Area}(B) \cdot h$



$V = \text{Area}(B_1) \cdot h$



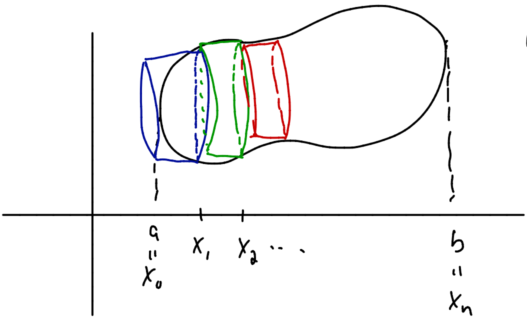


Cross-Section at x

Let $A(x)$ = area of the cross-section at x.

Let's approximate volume by breaking the solid S into n cylinders

$$\Delta x = \frac{b-a}{n}$$



Let's add up the volumes of the cylinders.

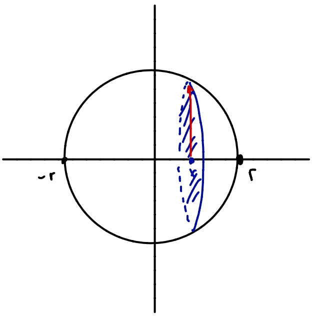
$$A(x_1)\cdot\Delta x + A(x_2)\cdot\Delta x + \cdots$$

$$= \sum_{j=1}^{n} A(x_n)\cdot\Delta x$$

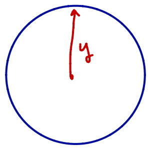
The more cylinders we take the better our approximation.

Def Let S be a solid that lies between $x=a$ and $x=b$. If the cross-sectional area of S in the plane through x and perpendicular to the x-axis is $A(x)$, then the volume of S is
$$V = \lim_{n \to \infty} \sum_{j=1}^{n} A(x_j)\,\Delta x = \int_a^b A(x)\,dx$$

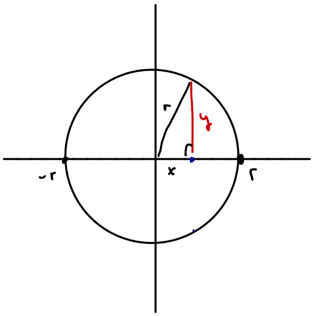
Ex Compute the volume of a sphere of radius r.



The cross-section at x is a circle.



We need to find its radius.

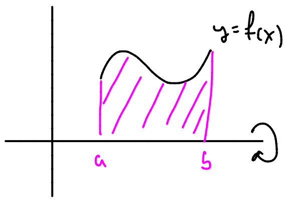


$$A(x) = \pi y^2 = \pi(r^2 - x^2)$$

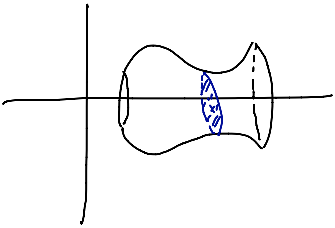
$$V = \int_{-r}^{r} \pi(r^2 - x^2)\,dx = \pi\int_{-r}^{r}(r^2 - x^2)\,dx = \pi\left(r^2x - \frac{x^3}{3}\right)\Big|_{-r}^{r}$$
$$= \pi\left(r^3 - \frac{r^3}{3}\right) - \pi\left(-r^3 + \frac{r^3}{3}\right)$$
$$= \pi\left(\frac{2}{3}r^3\right) + \pi\left(\frac{2}{3}r^3\right)$$
$$= \frac{4}{3}\pi r^3$$

Solids of Revolution

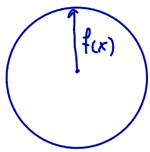
Take a region in the plane



and revolve around the x-axis to obtain a solid



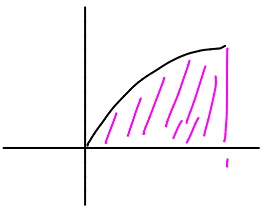
The cross section at x is

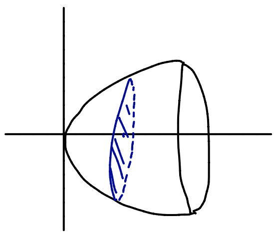


a disk of radius f(x)

So, $A(x) = \pi (f(x))^2$ and $V = \int_a^b \pi (f(x))^2 \, dx$

Ex Find the volume of the solid obtained by rotating about the x-axis the region bounded by $y=\sqrt{x}$, $x=0$, $x=1$, $y=0$





$$V = \int_0^1 \pi (\sqrt{x})^2 \cdot dx = \int_0^1 \pi x \, dx = \frac{\pi}{2}$$