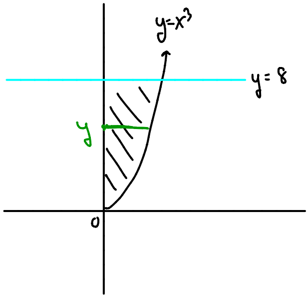
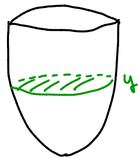
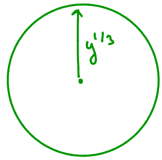
Ex Find the area of the solid obtained by revolving the region bounded by $y=x^3$, $y=8$, and $x=0$ about the y-axis.





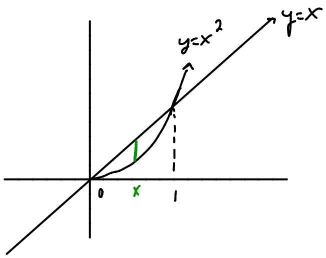


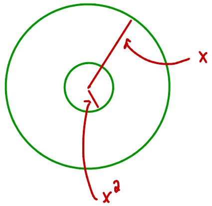
$A(y) = \pi y^{2/3}$

$$V = \int_0^8 \pi y^{2/3}\, dy = \frac{3\pi}{5} y^{5/3} \Big|_0^8 = \frac{96\pi}{5}$$

$$\boxed{\begin{array}{l} \text{Rotate about x-axis} \Rightarrow \text{Integrate wrt } x \\ \text{" \quad " \quad y-axis} \Rightarrow \text{" \quad " \quad } y \end{array}}$$

Ex Find the volume of the solid obtained by revolving the region bounded by $y=x$ and $y=x^2$ about the x-axis.



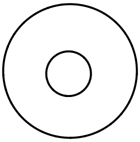


$y=x$ is the outer radius

$y=x^2$ is the inner radius

$$A(x) = \pi(x)^2 - \pi(x^2)^2$$
$$= \pi(x^2 - x^4)$$

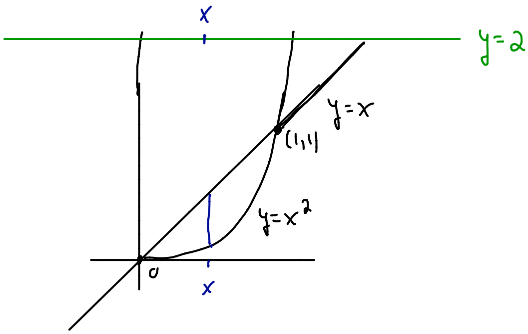
$$V = \int_0^1 \pi(x^2 - x^4)\,dx = \pi\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1 = \pi\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{2\pi}{15}$$

We call  a <u>washer</u>.

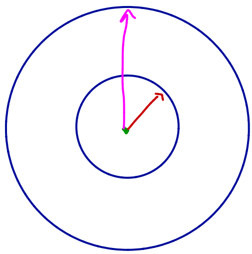
For a solid of revolution whose cross-sections are washers,

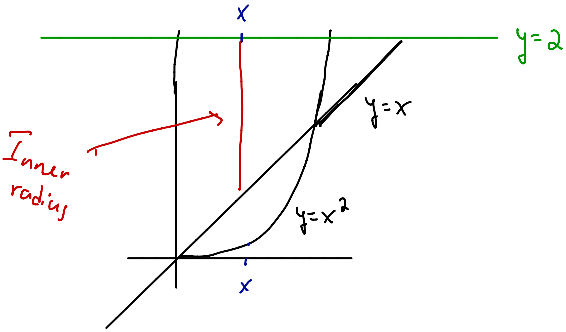
$$A = \pi(\text{outer radius})^2 - \pi(\text{inner radius})^2$$

<u>Ex.</u> Find the volume of the solid obtained by revolving the region bounded by $y=x$ and $y=x^2$ about the line $y=2$



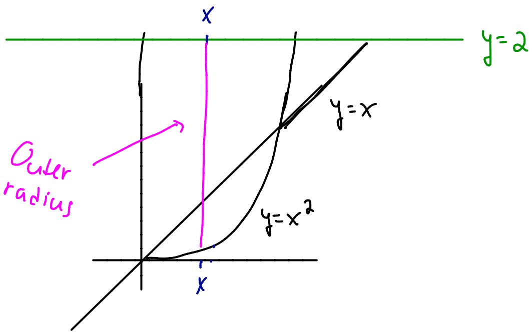
Cross Section at x





The inner radius is

$2-x$



The outer radius is $2-x^2$.

The volume is $\int_0^1 \left[\pi(\text{outer radius})^2 - \pi(\text{inner radius})^2\right] dx$

$$= \int_0^1 \left[\pi(2-x^2)^2 - \pi(2-x)^2\right] dx$$

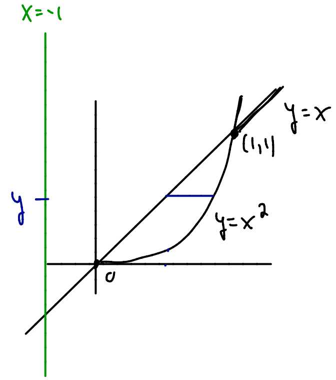
$$= \pi\int_0^1 \left[4-4x^2+x^4-(4-4x+x^2)\right] dx$$

$$= \pi\int_0^1 (x^4-5x^2+4x)\, dx$$

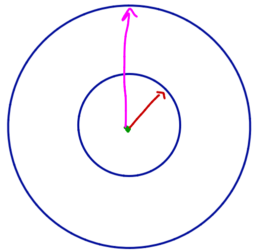
$$= \pi\left[\frac{x^5}{5}-\frac{5}{3}x^3+2x^2\right]\Bigg|_0^1$$

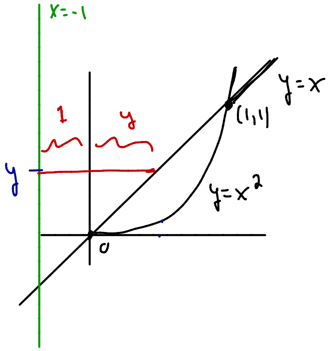
$$= \frac{8\pi}{15}$$

Ex. Find the volume of the solid obtained by revolving the region bounded by $y=x$ and $y=x^2$ about the line $x=-1$



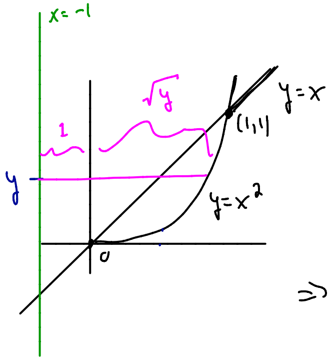
Cross Section at y





The inner radius is

$1+y$



The outer radius is

$1+\sqrt{y}$

$$\Rightarrow V=\int_0^1 \pi\left[(1+\sqrt{y})^2-(1+y)^2\right]dy$$