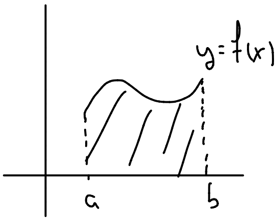
More generally, the volume of the solid obtained by revolving the region bounded by the curves $y=f(x)$, $y=0$, $x=a$, and $x=b$

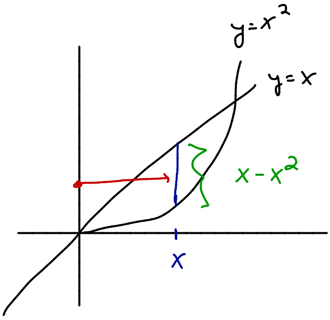


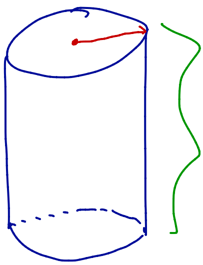

has volume $V=\int_a^b 2\pi x f(x)\,dx$

where $0 \le a < b$

In this formula, $x$ is playing the role of the radius of the shell and $f(x)$ the height of the shell.

Ex Find the volume of the solid obtained by rotating the region bounded by $y=x$ and $y=x^2$ about the $y$-axis.

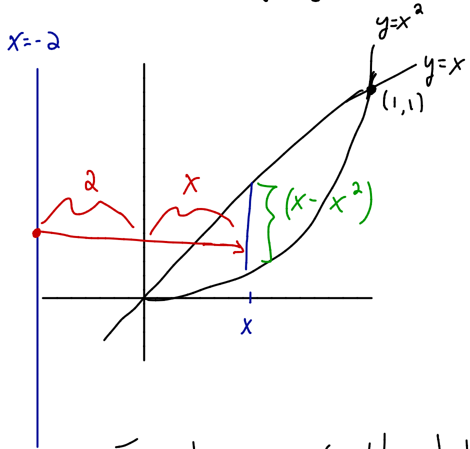


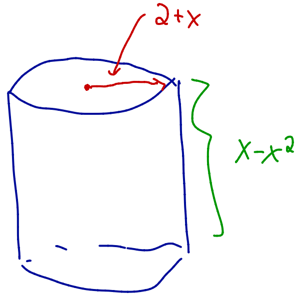



The radius is $x$.
The height is $x-x^2$.

$$V=\int_0^1 2\pi x(x-x^2)\,dx$$
$$=2\pi\int_0^1 (x^2-x^3)\,dx$$

Ex Find the volume of the solid obtained by rotating the region bounded by $y=x$ and $y=x^2$ about the line $x=-1$



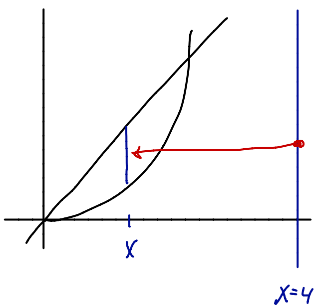




The height of the shell at $x$ is $x-x^2$

and the radius is $2+x$.

$$\Rightarrow V = \int_0^1 2\pi(2+x)(x-x^2)\,dx$$

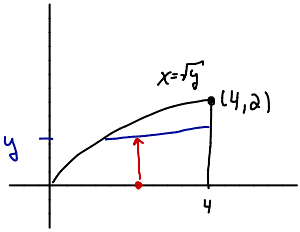
Ex Find the volume of the solid obtained by rotating the region bounded by $y=x$ and $y=x^2$ about the line $x=4$.

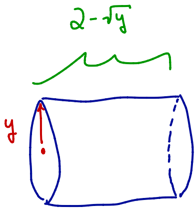



The height of the shell at $x$ is still $x-x^2$; but the radius is $5-x$.

$$\Rightarrow V = \int_0^1 2\pi(5-x)(x-x^2)\,dx$$

Ex. Find the volume of the solid obtained by revolving the region bounded by $x=y^3$, $y=0$, $x=0$, and $x=4$ about the x-axis.



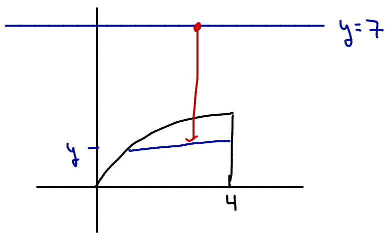




The shells in this case are indexed by y values, so everything should be in terms of y.

The height of the shell at y is $2-\sqrt{y}$ and the radius is y.

$$\Rightarrow V = \int_0^2 2\pi y\,(2-\sqrt{y})\,dy$$

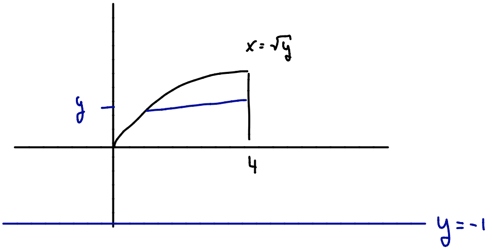
Ex. Find the volume of the solid obtained by revolving the region bounded by $x=y^3$, $y=0$, $x=0$, and $x=4$ about the line $y=7$




The height of the shell is still $(2-\sqrt{y})$, but the radius is now $7-y$.

$$\Rightarrow V = \int_0^2 2\pi(7-y)(2-\sqrt{y})\,dy$$

Ex. Find the volume of the solid obtained by revolving the region bounded by $x = y^3$, $y = 0$, $x = 0$, and $x = 4$ about $y = -1$




Again the height of the shell at $y$ is $2 - \sqrt{y}$. This time the radius is $1 + y$.

$$\Rightarrow V = \int_0^2 2\pi(1+y)(2-\sqrt{y})\,dy$$

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Solids of revolution overview:

Washer Method:

| Line is ... | Integrate wrt |
|---|---|
| Horizontal | $x$ |
| Vertical | $y$ |

Cylindrical Shell Method:

| Line is ... | Integrate wrt |
|---|---|
| Horizontal | $y$ |
| Vertical | $x$ |