# Antiderivates

**Def** An antiderivative of a function $f$ is a function $F$ such that $F' = f$.

**Ex**

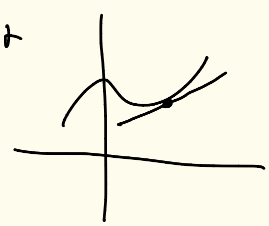
- $F(x) = \frac{1}{2}x^2$ is an antiderivative of $f(x) = x$.
- $F(x) = \sin x$ ———— " ———— $f(x) = \cos x$.
- (Ask class to produce at least 2 more antiderivatives of $f(x) = \cos x$.)

(Ask class what functions have $0$ as a derivative.)

**Theorem** If $f' = 0$, then $f$ is a constant function (that is, $f(x) = c$ for some constant $c$)

Reason: $f' = 0$ means $f$ can never increase or decrease.

Remember: derivative = slope of tangent



**Theorem** If $F$ and $G$ are antiderivatives of $f$, then there is some constant $c$ so that $G = F + c$.

Reason: $F' = G' = f$, so $(G - F)' = 0$.

$\Rightarrow G - F = c$ for some constant $c$.

$\Rightarrow G = F + c$.

Notation: The (general) antiderivative of a function $f(x)$ is written $F(x) + C$, where $F(x)$ is any antiderivative of $f$ and $C$ is an arbitrary constant.

To know an exact antiderivative of a function $f$, then you need to know the value of $f$ at some number.

Ex. · Find the antiderivative $F(x)$ of $f(x) = 2x$ with $F(1) = 10$.

$$F(x) = x^2 + C$$

$$F(1) = 10 \text{ and } F(1) = 1^2 + C$$

$$\Rightarrow 10 = 1 + C$$

$$\Rightarrow C = 9 \text{ and } F(x) = x^2 + 9.$$

· Find the antiderivative $F(x)$ of $f(x) = \sin(x)$ w/ $F(0) = 0$.

(Answer: $F(x) = -\cos x + 1$)

Table:

| Function | General antiderivative |
|---|---|
| $x^n$ $(n \neq 1)$ | $\frac{x^{n+1}}{n+1} + C$ |
| $\cos x$ | $\sin x + C$ |
| $\sin x$ | $-\cos x + C$ |
| $\sec^2 x$ | $\tan x + C$ |
| $\sec x \tan x$ | $\sec x + C$ |
| $a f(x)$ | $aF(x) + C$, $F(x)$ any a.d. of $f$. |
| $f(x) + g(x)$ | $F(x) + G(x)$, $F(x)$ and $G(x)$ any a.d.'s of $f$ and $g$, resp. |

Example: · Find $f$ if $f'(x) = 4\sqrt{x} + \sec^2(5x)$ and $f(0) = 3$

$\left( A: \frac{8}{3}x^{3/2} + \frac{1}{5}\tan(5x) + 3 \right)$

· Find all functions $f(x)$ so that $f''(x) = 3x^2 + 9x - 7$.

$$\left( \begin{aligned} A:\ & f' = x^3 + \tfrac{9}{2}x^2 - 7x + C \\ & f'' = \tfrac{1}{4}x^4 + \tfrac{3}{2}x^3 - \tfrac{7}{2}x^2 + Cx + D \end{aligned} \right)$$

## Rectilinear Motion

Recall: If $s(t)$ is the displacement of an object moving in a line with respect to time, then

- $v(t) = s'(t)$ is the objects velocity, and
- $a(t) = v'(t) = s''(t)$ is its acceleration.

Example If a ball is thrown directly upwards at 70 ft/s from the edge of Kiely Hall 150 ft above ground, then

a) what is its maximum height?

b) What is its velocity when it hits the ground?

Solution: First need to find $s(t)$ and $v(t)$.

What do we know? $a(t) = -32\ ft/s^2$ (acceleration due to gravity)

So, $v(t) = -32t + C$ and $v(0) = 70$

$\Rightarrow v(t) = -32t + 70$

Now, $s(t) = -16t^2 + 70t + C'$ and $s(0) = 150$

$\Rightarrow s(t) = -16t^2 + 70t + 150$

And, now calculate... (Need to find $t$ when $v(t) = 0$ and when $s(t) = 0$)