# Homework 11 MATH 231

Due Wednesday, November 30, 2023

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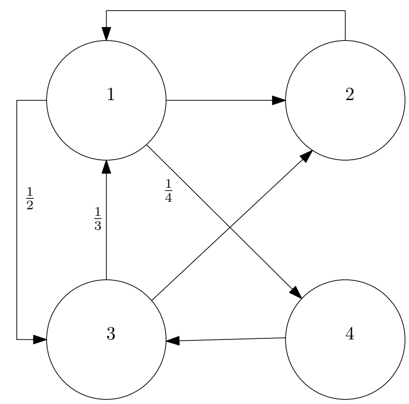
**Instructions.** *Your work will be collected in class on the due date. We will also have a quiz in class on the due date based on the content from the assignment.*




**Exercise 1.** Pictured to the right is a Markov chain.
(a) Fill in the missing edge labels.
(b) Write down the transition matrix $M$ for the Markov chain.
(c) Convince yourself that the Markov chain is regular.
(d) Find the unique stochastic vector $\mathbf{w}$ such that $M\mathbf{w} = \mathbf{w}$.

**Exercise 2.** Consider the (incomplete) matrix:

$$A = \begin{bmatrix} \frac{1}{3} & * & 0 & \frac{2}{9} & 0 \\ 0 & 0 & 0 & 0 & * \\ * & \frac{2}{7} & 0 & * & 0 \\ 0 & \frac{3}{7} & 0 & 0 & \frac{1}{5} \\ \frac{1}{6} & 0 & * & 0 & 0 \end{bmatrix}$$

(a) Replace each asterisk mark in $A$ with a real number so that the result is a stochastic matrix.

(b) Draw the Markov chain associated to $A$.

(c) Determine if the underlying directed graph is strongly connected.

**Exercise 3.** Draw a Markov chain representing the following situation: *The weather in Edinburgh is either good, indifferent, or bad on any given day. If the weather is good today, there is a 50% chance the weather will be good tomorrow, a 30% chance the weather will be indifferent, and a 20% chance the weather will be bad. If the weather is indifferent today, it will be good tomorrow with probability .20 and indifferent with probability .70. Finally, if the weather is bad today, it will be good tomorrow with probability .10 and indifferent with probability .30.*

**Exercise 4.** Let $M$ be an $n \times n$ stochastic matrix, and let $\mathbf{w} \in \mathbb{R}^n$ be a stochastic vector. Show that $M\mathbf{w}$ is stochastic.

**Exercise 5.** Complete the following exercises from Section 6.1 in the course textbook:

# 1, 3, 5, 9, 13, 15, 33, 35, 37, 38, 39

**Exercise 6.** Let $W$ be a subspace of $\mathbb{R}^n$. Show that $W \subset (W^{\perp})^{\perp}$.