Quiz 1

Wednesday, September 6, 2023

MATH 231

Spring 2023

Problem 1. Consider the following matrix:

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 2 \end{bmatrix}$$

(a) Explain why the above matrix is not in *reduced* row echelon form.

In order to be in RREF, any pivot column can have only one nonzero entry. The second column of the above matrix is a pivot column, but has two nonzero entries, so it is not in RREF.

(b) Use a single elementary row operation to find the reduced row echelon form of the above matrix and make clear what the operation is.

Subtract 3 times the second row from the first row:

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$$

Problem 2. Construct three different augment matrices for linear systems whose solution set is $x_1 = 5$, $x_2 = -1$, and $x_3 = 1$. Briefly explain how you know they all have the same solution set.

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$ is the augmented matrix for the system $x_1 = 5$, $x_2 = -1$, $x_3 = 1$, which clearly has the desired solution.

Now perform any elementary row operation to the above matrix to get a new aug. matrix for a system w/ the same solutions

e.g. $\begin{bmatrix} 2 & 0 & 0 & 10 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \leftarrow 2R_1$

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$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 1 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix} \leftarrow R_2 + R_1$

Problem 3. Given below is the reduced row echelon form of an augmented matrix associated to a linear system in the variables x_1, x_2, x_3, and x_4.

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 5 \\ 0 & 1 & 2 & 0 & -3 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(a) List the basic variable(s).

X_1, X_2, X_4

(b) List the free variable(s).

X_3

(c) Write down the general solution to the linear system.

The linear system for the above matrix is

$X_1 + 3X_3 = 5$

$X_2 + 2X_3 = -3$

$X_4 = 0$.

Assign parameter t to X_3 to get the general solution

$X_1 = -3t + 5$

$X_2 = -2t - 3$

$X_3 = t$

$X_4 = 0$.

Where $t \in \mathbb{R}$.

(d) How many solutions does the linear system have?

∞-many.

There is a solution for every $t \in \mathbb{R}$.