

Name: Solutions

Quiz 3

Wednesday, September 20, 2023

MATH 231

Spring 2023

Problem 1. Let  $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 2 \\ 7 & 1 & 8 \end{bmatrix}$  and let  $b = \begin{bmatrix} -2 \\ 10 \\ 54 \end{bmatrix}$ .

(a) Express the matrix equation  $Ax = b$  as a system of linear equations.

$$x_1 - x_2 = -2$$

$$2x_2 + 2x_3 = 10$$

$$7x_1 + x_2 + 8x_3 = 54$$

(b) Use the fact that

$$\text{rref} \left( \begin{bmatrix} 1 & -1 & 0 & -2 \\ 0 & 2 & 2 & 10 \\ 7 & 1 & 8 & 54 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

to determine if  $Ax = b$  has a solution. Give a brief explanation of your reasoning.

No, the last column is a pivot column.

Alternatively, the last row reads  $0 = 1$ ,  
which is inconsistent.

(c) Give the three vectors  $a_1, a_2$ , and  $a_3$  in  $\mathbb{R}^3$  so that  $A = [a_1 \ a_2 \ a_3]$ .

$$a_1 = \begin{bmatrix} 1 \\ 0 \\ 7 \end{bmatrix}, \quad a_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \quad a_3 = \begin{bmatrix} 0 \\ 2 \\ 8 \end{bmatrix}$$

(d) Is  $b$  in  $\text{span}\{a_1, a_2, a_3\}$ ? Explain.

No; ~~since~~

if  $b = c_1 a_1 + c_2 a_2 + c_3 a_3$ ,  
(Turn page over.)

then  $A \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = b$ , but

$Ax = b$  does not have  
a solution.

Problem 2. Let  $A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 3 & 1 \end{bmatrix}$

(a) Is  $v = \begin{bmatrix} 6 \\ -1 \\ 3 \end{bmatrix}$  a solution to  $Ax = 0$ ? (Show your computation.)

$$Av = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 + 0 + -6 \\ 0 - 3 + 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

Yes.

(b) Does the linear system corresponding to  $Ax = 0$  have a free variable? Explain.

Yes, since  $v = \begin{bmatrix} 6 \\ -1 \\ 3 \end{bmatrix}$  is a <sup>nontrivial</sup> solution to  $Ax = 0$ ,

$Ax = 0$  must have a free variable.