

Name:

Quiz 6

Wednesday, October 18, 2023

MATH 231

Fall 2023

Instructions: Take this quiz at home under the same conditions as in class: no notes, no help from anyone else, and no asking the internet. If you do not want to print the quiz, you may complete it on a blank sheet of paper, just make sure to label your questions clearly.

Problem 1. Let $A = \begin{bmatrix} 1 & -2 & 9 & 5 & 4 \\ 1 & -1 & 6 & 5 & -3 \\ -2 & 0 & -6 & 1 & -2 \\ 4 & 1 & 9 & 1 & -9 \end{bmatrix}$, and suppose A is row equivalent to the following matrix in row echelon form:

$$\begin{bmatrix} 1 & -2 & 9 & 5 & 4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Find a basis for the null space of A .

(b) Find a basis for the column space of A .

(c) Find the rank and nullity of A .

(Turn page over.)

Problem 2. Let A be a 4×6 matrix.

(a) What is the smallest possible dimension for the null space of A ? Briefly explain how you know.

(b) If A has a single pivot column, what is the dimension of the null space of A ? Briefly explain how you know.

(c) If the equation $Ax = 0$ has 2 free variables, what is the rank and nullity of A ? Briefly explain how you know.

(d) If $Ax = b$ is consistent for every $b \in \mathbb{R}^4$, then what are the rank and nullity of A ? Briefly explain how you know.