

Name:

Quiz 6

Wednesday, October 18, 2023

MATH 231

Fall 2023

Instructions: Take this quiz at home under the same conditions as in class: no notes, no help from anyone else, and no asking the internet. If you do not want to print the quiz, you may complete it on a blank sheet of paper, just make sure to label your questions clearly.

Problem 1. Let $A = \begin{bmatrix} 1 & -2 & 9 & 5 & 4 \\ 1 & -1 & 6 & 5 & -3 \\ -2 & 0 & -6 & 1 & -2 \\ 4 & 1 & 9 & 1 & -9 \end{bmatrix}$, and suppose A is row equivalent to the following matrix in row echelon form:

$$\begin{bmatrix} 1 & -2 & 9 & 5 & 4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 9 & 0 & 14 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis for the null space of A .

$$\text{ref}(A) = \begin{bmatrix} 1 & -2 & 9 & 0 & 14 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \rightsquigarrow \text{null}(A) = \left\{ \begin{bmatrix} -3s - 7t \\ 3s + 7t \\ s \\ 2t \\ t \end{bmatrix} \mid s, t \in \mathbb{R} \right\}$$
$$= \text{Span} \left\{ \begin{bmatrix} -3 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 7 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$$

(b) Find a basis for the column space of A .

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix} \right\} \text{ is a basis for } \text{col}(A) \text{ since they are the pivot columns of } A.$$

(c) Find the rank and nullity of A .

$$\text{rank}(A) = \dim(\text{col}(A)) = 3$$
$$\text{nullity}(A) = \dim(\text{null}(A)) = 2$$

(Turn page over.)

Problem 2. Let A be a 4×6 matrix.

- (a) What is the smallest possible dimension for the null space of A ? Briefly explain how you know.

A has at most 4 pivot columns; and
hence $Ax=0$ must have at least two free
variables $\Rightarrow \text{nullity}(A) \geq 2$.

- (b) If A has a single pivot column, what is the dimension of the null space of A ? Briefly explain how you know.

$Ax=0$ then has 5 free variables
 $\Rightarrow \text{nullity}(A) = 5$.

- (c) If the equation $Ax = 0$ has 2 free variables, what is the rank and nullity of A ? Briefly explain how you know.

$$\text{nullity}(A) = 2$$

A has 4 pivot columns since it has 4 leading
variables $\Rightarrow \text{rank}(A) = 4$

- (d) If $Ax = b$ is consistent for every $b \in \mathbb{R}^4$, then what are the rank and nullity of A ? Briefly explain how you know.

This says every row of A has a pivot
position $\Rightarrow \text{rank}(A) = 4$
 $\Rightarrow Ax=0$ has 2 free variables
 $\Rightarrow \text{nullity}(A) = 2$.