Name: Solutions

Quiz 7

Wednesday, October 25, 2023

MATH 231
Fall 2023

Problem 1. Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$. Find $\mathbf{x} \in \mathbb{R}^2$ satisfying $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.

$$X = 3\begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$$

Problem 2. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -3 \\ -7 \\ 5 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 4 \\ 10 \\ -7 \end{bmatrix}$.

(a) Explain how you know that $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent.

V_2 is not a multiple of V_1.

(b) Let $H = \operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2\}$. By part (a), $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ is a basis for H. Use that the reduced row echelon form of the matrix $[\mathbf{v}_1\ \mathbf{v}_2\ \mathbf{b}]$ is $\begin{bmatrix} 1 & 0 & \frac{1}{4} \\ 0 & 1 & -\frac{5}{4} \\ 0 & 0 & 0 \end{bmatrix}$ to find the $\mathcal{B}$-coordinates of $\mathbf{b}$, that is, compute $[\mathbf{b}]_{\mathcal{B}}$.

The solution to $X_1V_1 + X_2V_2 = b$

is $X_1 = \frac{1}{4}$

$X_2 = -5/4$

Since $[v_1\ v_2\ b]$ is the augmented matrix for the associated linear system. So, $[b]_{\mathcal{B}} = \begin{bmatrix} 1/4 \\ -5/4 \end{bmatrix}$.

(Turn page over.)

Problem 3. Let $A = \begin{bmatrix} 1 & -3 & 2 & -4 \\ -3 & 9 & -1 & 5 \\ 2 & -6 & 4 & -3 \\ -4 & 12 & 2 & 7 \end{bmatrix}$. Then, A is row equivalent to the matrix $\begin{bmatrix} 1 & -3 & 2 & -4 \\ 0 & 0 & 5 & -7 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Give a basis for the column space of A.

$$\left\{ \begin{bmatrix} 1 \\ -3 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \\ -3 \\ 7 \end{bmatrix} \right\}$$

(b) Give a basis for the null space of A.

$$A \sim \begin{bmatrix} 1 & -3 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{null}(A) = \left\{ \begin{bmatrix} 3t \\ t \\ 0 \\ 0 \end{bmatrix} \middle| \, t \in \mathbb{R} \right\}$$

$$= \text{Span}\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$$\Rightarrow \left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\} \text{ is a basis for null}(A).$$