Name: Solutions

Quiz 8
Wednesday, November 1, 2023

MATH 231
Fall 2023

Problem 1. Compute the determinant of each of the following matrices. Show your work: you cannot simply enter it in your calculator.

(a) $\begin{bmatrix} 2 & -2 & 3 \\ 3 & 1 & 2 \\ 1 & 3 & -1 \end{bmatrix}$

$$\det(\quad) = 2\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} - (-2)\begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix} + 3\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix}$$

$$= 2(-7) + 2(-5) + 3(8)$$

$$= -14 - 10 + 24$$

$$= 0$$

(b) $\begin{bmatrix} 4 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 3 & 0 & 0 & 0 \\ 8 & 3 & 1 & 7 \end{bmatrix}$

$$\det(\quad) = 3\begin{vmatrix} 0 & 0 & 5 \\ 7 & 2 & -5 \\ 3 & 1 & 7 \end{vmatrix} = 3 \cdot 5 \begin{vmatrix} 7 & 2 \\ 3 & 1 \end{vmatrix} = 15(1) = 15$$

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Problem 2. Let $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$. Use your work from Problem 1(a) to decide whether $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set. Explain your reasoning.

Let $A = [v_1 \ v_2 \ v_3]$. Then $\det(A) = 0$ by 1(a).

$\Rightarrow$ The columns of A, and hence $\{v_1, v_2, v_3\}$, are linearly dependent.

Problem 3. Let A and B be 3×3 matrices, with $\det(A) = 2$ and $\det(B) = 3$. Use properties of the determinant to compute the following:

(a) $\det(5A)$ $= 5^3 \cdot \det(A) = 250$

(b) $\det(AB)$ $= \det(A)\det(B) = 6$

(c) $\det(B^2)$ $= \det(BB) = \det(B)\det(B) = 9$

(d) $\det(ABA^{-1})$ $= \det(A)\det(B)\det(A^{-1}) = \det(A)\det(B)\frac{1}{\det(A)}$

$= \det(B) = 3$