Quiz 10

Wednesday, May 3, 2023

MATH 231
Spring 2023

Problem 1. Let W be a subspace of $\mathbb{R}^n$, and let $\{\mathbf{w}_1, \mathbf{w}_2\}$ be a basis for W. Show that if $\mathbf{v} \cdot \mathbf{w}_1 = 0$ and $\mathbf{v} \cdot \mathbf{w}_2 = 0$, then $\mathbf{v} \in W^{\perp}$.

Let $w \in W$. Then $\exists\, c_1, c_2 \in \mathbb{R}$ such that $w = c_1 w_1 + c_2 w_2$.

So, $v \cdot w = c_1(\underbrace{v \cdot w_1}_{0}) + c_2(\underbrace{v \cdot w_2}_{0}) = 0$

As w was arbitrary, $v \in W^{\perp}$.

Problem 2. Let $\mathbf{u}_1 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ and let $\mathbf{u}_2 = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$.

1. Show that $\{\mathbf{u}_1, \mathbf{u}_2\}$ is an orthogonal basis for $\mathbb{R}^2$.

$u_1 \cdot u_2 = 2 \cdot 6 + (-3)4 = 0$

$\Rightarrow u_1, u_2$ are orthogonal $\Rightarrow \{u_1, u_2\}$ is linearly ~~indep~~ independent, and hence a basis as $\dim \mathbb{R}^2 = 2$.

$\Rightarrow \{u_1, u_2\}$ is an orthogonal basis for $\mathbb{R}^2$.

2. Express $\mathbf{v} = \begin{bmatrix} 9 \\ -7 \end{bmatrix}$ as a linear combination in $\mathbf{u}_1$ and $\mathbf{u}_2$.

$$v = \left(\frac{v \cdot u_1}{u_1 \cdot u_1}\right) u_1 + \left(\frac{v \cdot u_2}{u_2 \cdot u_2}\right) u_2$$

$$= \frac{39}{13} u_1 + \left(\frac{26}{52}\right) u_2$$

$$= 3u_1 + \tfrac{1}{2} u_2$$