

Quiz 11

Wednesday, May 10, 2023

MATH 231

Spring 2023

Problem 1. Let $\mathbf{y} = \begin{bmatrix} -1 \\ 2 \\ 6 \end{bmatrix}$, $\mathbf{u}_1 = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$, and $\mathbf{u}_2 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$.

(a) Verify that $\{\mathbf{u}_1, \mathbf{u}_2\}$ is an orthogonal set (show your computation).

$$\mathbf{u}_1 \cdot \mathbf{u}_2 = 3(1) + (-1)(-1) + 2(-2)$$

$$= 3 + 1 - 4$$

$$= 0$$

$\Rightarrow \mathbf{u}_1$ and \mathbf{u}_2 are orthogonal

(b) Find the orthogonal projection of \mathbf{y} onto $\text{span}\{\mathbf{u}_1, \mathbf{u}_2\}$.

$$\begin{aligned} \text{proj}_{\text{span}\{\mathbf{u}_1, \mathbf{u}_2\}}(\mathbf{y}) &= \frac{\mathbf{y} \cdot \mathbf{u}_1}{\mathbf{u}_1 \cdot \mathbf{u}_1} \mathbf{u}_1 + \frac{\mathbf{y} \cdot \mathbf{u}_2}{\mathbf{u}_2 \cdot \mathbf{u}_2} \mathbf{u}_2 \\ &= \frac{7}{14} \mathbf{u}_1 + \frac{-15}{6} \mathbf{u}_2 = \begin{bmatrix} -1 \\ 2 \\ 6 \end{bmatrix} \end{aligned}$$

Problem 2. Let $\mathbf{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, and let $W = \text{span}\{\mathbf{v}\}$. Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be given by

$$T(\mathbf{u}) = \text{proj}_W(\mathbf{u})$$

so that T is a linear transformation. Find the matrix A satisfying $T(\mathbf{u}) = A\mathbf{u}$ for every $\mathbf{u} \in \mathbb{R}^2$.

$$A = [T(\mathbf{e}_1) \quad T(\mathbf{e}_2)]$$

$$\begin{aligned} T(\mathbf{e}_1) &= \frac{\mathbf{e}_1 \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}} \cdot \mathbf{v} & T(\mathbf{e}_2) &= \frac{\mathbf{e}_2 \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}} \mathbf{v} \\ &= \frac{3}{25} \mathbf{v} & &= \frac{4}{25} \mathbf{v} \end{aligned}$$

$$\Rightarrow A = \frac{1}{25} \begin{bmatrix} 9 & 12 \\ 12 & 16 \end{bmatrix}$$