Quiz 2 MATH 231

Wednesday, February 15, 2021 Spring 2023

Problem 1. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 2 \\ 7 & 1 & 8 \end{bmatrix}$ and let $\mathbf{b} = \begin{bmatrix} -2 \\ 10 \\ 54 \end{bmatrix}$.

(a) Express the matrix equation $A\mathbf{x} = \mathbf{b}$ as a vector equation.

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 7 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 2 \\ 8 \end{bmatrix} = \begin{bmatrix} -2 \\ 10 \\ 54 \end{bmatrix}$$

(b) Express the matrix equation $A\mathbf{x} = \mathbf{b}$ as a system of linear equations.

$$x_1 - x_2 = -2$$
$$2x_2 + 2x_3 = 10$$
$$7x_1 + x_2 + 8x_3 = 54$$

(c) Use the fact that

$$\text{rref}\left(\begin{bmatrix} 1 & -1 & 0 & -2 \\ 0 & 2 & 2 & 10 \\ 7 & 1 & 8 & 54 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

to determine if $A\mathbf{x} = \mathbf{b}$ has a solution. Give a brief explanation of your reasoning.

No, because the rightmost column is a pivot column (which says $0 = 1$, a contradiction)

(d) Give the three vectors $\mathbf{a}_1, \mathbf{a}_2$, and $\mathbf{a}_3$ in $\mathbb{R}^3$ so that $A = [\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3]$.

$$a_1 = \begin{bmatrix} 1 \\ 0 \\ 7 \end{bmatrix}, \quad a_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \quad a_3 = \begin{bmatrix} 0 \\ 2 \\ 8 \end{bmatrix}$$

(Turn page over.)

(e) Is $\mathbf{b}$ in span$\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$? Explain.

No, since the vector equation

$$x_1 a_1 + x_2 a_2 + x_3 a_3 = b$$

has no solution.

(f) Is $\mathbf{a}_3$ in span$\{\mathbf{a}_1, \mathbf{a}_2\}$? (Hint: You can read off the matrix rref(A) from the above rref matrix.)

$a_3 \in \text{span}\{a_1, a_2\} \Leftrightarrow x_1 a_1 + x_2 a_2 = a_3$ has a solution. The associated aug. matrix is A and $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow$ Yes $a_3 \in \text{span}\{a_1, a_2\}$

Problem 2. Let A be a 3×2 matrix. Explain why the matrix equation $A\mathbf{x} = \mathbf{b}$ *cannot* be consistent for every $\mathbf{b} \in \mathbb{R}^3$.

$Ax = b$ is consistent $\forall b \Leftrightarrow$ every row of A contains a leading 1

A has only two columns and hence at most 2 leading ones, but 3 rows $\Rightarrow Ax = b$ is <u>not</u> consistent $\forall b \in \mathbb{R}^3$.

Problem 3. Let $A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 3 & 1 \end{bmatrix}$. Is $\mathbf{v} = \begin{bmatrix} 6 \\ 1 \\ 3 \end{bmatrix}$ a solution to $A\mathbf{x} = 0$? (Show your computation.)

$$Av = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix} \neq 0.$$

No, v is not a solution to $Ax = 0$.