Quiz 3 MATH 231
Wednesday, February 22, 2023 Spring 2023

Instructions: This is a take-home quiz. Complete the following problems (on any paper you like) and turn in your solutions in class Monday, February 27. *****You may use your textbook and your homework, but you may not use any other sources.**

Problem 1. Let A be an $m \times n$ matrix and let $\mathbf{b} \in \mathbb{R}^m$. Suppose $A\mathbf{x} = \mathbf{b}$ is consistent and that $\mathbf{v}$ and $\mathbf{w}$ are distinct solutions (that is, $\mathbf{v} \neq \mathbf{w}$). Give a nontrivial solution to the homogeneous equation $A\mathbf{x} = \mathbf{0}$. Justify your answer.

Problem 2. Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4 \in \mathbb{R}^4$, and let $A = [\mathbf{v}_1\ \mathbf{v}_2\ \mathbf{v}_3\ \mathbf{v}_4]$. Given that

$$\operatorname{rref}(A) = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

determine if the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are linearly independent or linearly dependent. Explain your answer.

(Turn page over.)

Problem 3. Let $\mathbf{v}_1, \ldots, \mathbf{v}_p$ be vectors in $\mathbb{R}^n$. Show that if $\mathbf{v}_p \in \text{span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_{p-1}\}$, then $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ is a subset of $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_{p-1}\}$.