Quiz 5

Wednesday, March 14, 2023 — MATH 231, Spring 2023

Problem 1. Find the inverse of $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$ (if it exists). Show your work.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{array}\right]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} 8 & 3 & 1 \\ 16 & 4 & 1 \\ 7/2 & 3/2 & 1/2 \end{bmatrix}$$

Problem 2. Is it possible for a $\underbrace{5 \times 5 \text{ matrix}}_{A}$ to be invertible when its columns do not span $\mathbb{R}^5$? Why or why not?

No. If its columns do not span $\mathbb{R}^5$, then there is some $b \in \mathbb{R}^5$ for which $Ax=b$ is inconsistent.

But if A is invertible then $A^{-1}b$ is a solution to $Ax=b$, contradicting the previous statement.

Problem 3. Let A be a matrix that is row equivalent to the matrix $\begin{bmatrix} 1 & 2 & 6 & -5 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Find a basis for the null space of A.

$$Ax=0 \Rightarrow \begin{aligned} x_1 &= -2x_2 - 6s + 5t = 4s - 7t \\ x_2 &= -5s + 6t \\ x_3 &= s \\ x_4 &= t \end{aligned}$$

$$\Rightarrow \operatorname{null}(A) = \left\{ s\begin{bmatrix} 4 \\ -5 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -7 \\ 6 \\ 0 \\ 1 \end{bmatrix} \;\middle|\; s,t \in \mathbb{R} \right\}$$

$$= \operatorname{span}\left\{ \begin{bmatrix} 4 \\ -5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 6 \\ 0 \\ 1 \end{bmatrix} \right\} \Rightarrow \mathcal{B} = \left\{ \begin{bmatrix} 4 \\ -5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 6 \\ 0 \\ 1 \end{bmatrix} \right\}$$

is a basis for null(A).