

## Quiz 6

Wednesday, March 22, 2023

MATH 231

Spring 2023

**Problem 1.** Let  $A = \begin{bmatrix} 1 & -2 & 9 & 5 & 4 \\ 1 & -1 & 6 & 5 & -3 \\ -2 & 0 & -6 & 1 & -2 \\ 4 & 1 & 9 & 1 & -9 \end{bmatrix}$ , and suppose  $A$  is row equivalent to the following matrix in row echelon form:

$$\begin{bmatrix} 1 & -2 & 9 & 5 & 4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Find a basis for the column space of  $A$ .

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 1 \\ 1 \end{bmatrix} \right\}$$

(b) Find the rank and nullity of  $A$ .

$$\text{rank}(A) = 3, \text{ nullity}(A) = 2$$

**Problem 2.** Let  $A$  be a  $4 \times 6$  matrix.

(a) What is the smallest possible dimension for the null space of  $A$ ? Briefly explain how you know.

$$\text{rank}(A) \leq 4 \Rightarrow \text{nullity}(A) \geq 2 \text{ by Rank thm} \\ \text{b/c } \text{col}(A) \text{ subspace of } \mathbb{R}^4$$

(b) If  $A$  has a single pivot column, what is the dimension of the null space of  $A$ ? Briefly explain how you know.

$$\text{rank}(A) = \# \text{ pivot columns} = 1 \Rightarrow \text{nullity}(A) = 5 \text{ by rank thm}$$

(c) If the equation  $Ax = 0$  has 2 free variables, what is the rank and nullity of  $A$ ? Briefly explain how you know.

$$\text{nullity}(A) = \# \text{ free variables} = 2 \Rightarrow \text{rank}(A) = 4 \text{ by rank thm}$$

(d) If  $Ax = b$  is consistent for every  $b \in \mathbb{R}^4$ , then what are the rank and nullity of  $A$ ? Briefly explain how you know.

$$Ax = b \text{ consistent } \forall b \in \mathbb{R}^4 \Rightarrow \text{col}(A) = \mathbb{R}^4 \Rightarrow \text{rank}(A) = 4$$

$$\Rightarrow \text{nullity}(A) = 2 \text{ by rank thm}$$