Quiz 7 MATH 231
Wednesday, March 29, 2023 Spring 2023

Problem 1. Let $A = \begin{bmatrix} 6 & -3 & 2 \\ 0 & 5 & -5 \\ 3 & -7 & 8 \end{bmatrix}$.

(a) Compute the determinant (by hand) of A.

1st column expansion:

$$\det(A) = 6\begin{vmatrix} 5 & -5 \\ -7 & 8 \end{vmatrix} - 0\begin{vmatrix} -3 & 2 \\ -7 & 8 \end{vmatrix} + 3\begin{vmatrix} -3 & 2 \\ 5 & -5 \end{vmatrix} = 6(5) + 3(5) = 45$$

(b) Based on your answer in part (a), is A invertible?

Yes

Problem 2. Suppose that A is a square matrix such that $\det(A^3) = 0$. Explain why A cannot be invertible.

$$0 = \det(A^3) = \det(A^2A) = \det(A)\det(A^2)$$
$$= \det(A)\det(A \cdot A) = (\det(A))^3 \Rightarrow \det(A) = 0$$

Problem 3. Let A and B be 4×4 matrices with $\det(A) = 5$ and $\det(B) = -3$. Use the properties of determinants to compute:

(a) $\det(AB)$

-15

(b) $\det(2A) = 2^4 \det(A) = 16 \cdot 5 = 80$

(c) $\det(A^T B^2) = \det A \cdot (\det(B))^2 = 5 \cdot 9 = 45$

(d) $\det(B^{-1}) = \frac{1}{\det B} = -\frac{1}{3}$

(e) $\det(C)$, where C is the matrix obtained by first multiplying the 2nd row of A by 3 and then swapping the first and last row of A.

$$\det(C) = (-1)(3)\det(A) = -15.$$