

## Quiz 8

Wednesday, April 19, 2023

MATH 231

Spring 2023

**Problem 1.** Let  $A = \begin{bmatrix} 2 & 7 \\ 7 & 2 \end{bmatrix}$ .

(a) Find the characteristic polynomial of  $A$ .

$$\begin{aligned} \det(\lambda I - A) &= \begin{vmatrix} \lambda - 2 & -7 \\ -7 & \lambda - 2 \end{vmatrix} = (\lambda - 2)^2 - 49 = \lambda^2 - 4\lambda - 45 \\ &= (\lambda - 9)(\lambda + 5) \end{aligned}$$

(b) From your answer in part (a), what are the eigenvalues of  $A$ ?

9 and -5

(c) The eigenvalues for  $A$  are -5 and 9 (you can use this to check your answer above). Find the eigenspace associated to each eigenvalue of  $A$ .

$$\begin{aligned} E_9 &= \text{null}(9I - A) \\ &= \text{null}\left(\begin{bmatrix} 7 & -7 \\ -7 & 7 \end{bmatrix}\right) \end{aligned}$$

$$\begin{aligned} \begin{bmatrix} 7 & -7 \\ -7 & 7 \end{bmatrix} &\sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \\ \Rightarrow E_9 &= \text{span}\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\} \end{aligned}$$

$$\begin{aligned} E_{-5} &= \text{null}(-5I - A) = \text{null}\left(\begin{bmatrix} -7 & -7 \\ -7 & -7 \end{bmatrix}\right) \\ \begin{bmatrix} -7 & -7 \\ -7 & -7 \end{bmatrix} &\sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \\ \Rightarrow E_{-5} &= \text{span}\left\{\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right\} \end{aligned}$$

(d) Is  $A$  diagonalizable? If so, provide a diagonal matrix that is similar to  $A$ .

Yes,  $\begin{bmatrix} 9 & 0 \\ 0 & -5 \end{bmatrix}$

(Turn page over.)

**Problem 2.** Suppose  $A$  is a square matrix such that  $A^2$  is the zero matrix.

(a) Explain how you know that 0 is an eigenvalues of  $A$ .

(b) Show that if  $\lambda$  is an eigenvalue of  $A$ , then  $\lambda = 0$ .