Theorem 1 (Invertible Matrix Theorem)**.** Let A be an $n \times n$ matrix. Then the following statements are equivalent.

a. A is an invertible matrix.

b. A is row equivalent to I_n.

c. A has n pivot positions.

d. The equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

e. The columns of A form a linearly independent set.

f. The linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one.

g. The equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b} \in \mathbb{R}^n$.

h. The columns of A span $\mathbb{R}^n$.

i. The linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ maps $\mathbb{R}^n$ onto $\mathbb{R}^n$.

j. There is an $n \times n$ matrix C such that $CA = I_n$.

k. There is an $n \times n$ matrix D such that $AD = I$.

l. A^T is an invertible matrix.

m. The columns of A form a basis of $\mathbb{R}^n$.

n. $\text{col}(A) = \mathbb{R}^n$.

o. $\text{rank}(A) = n$.

p. $\text{nullity}(A) = 0$.

q. $\text{null}(A) = \{\mathbf{0}\}$.

r. $\det(A) \neq 0$.