Homework 2

MATH 301

Solution to graded problem

Exercise 5. Let $n \in \mathbb{N}$. Prove that given any $m \in \mathbb{Z}$ there exists a unique element $a \in \{0, 1, 2, \ldots, n-1\}$ such that $m \equiv a \pmod{n}$.

Solution. Let $m \in \mathbb{Z}$. By the division algorithm, there exists unique $q, a \in \mathbb{Z}$ such that $m = qn + a$ and $a \in \{0, 1, \ldots, n-1\}$. Rearranging the above equality, we have that $m - a = qn$, and hence $n \mid m - a$. This implies that $m \equiv a \pmod{n}$ and $a \in \{0, 1, \ldots, n-1\}$, as desired. It is left to show that a is unique: let $a' \in \{0, 1, \ldots, n-1\}$ such that $m \equiv a' \pmod{n}$. Similar to the above (but in reverse), there exists $q' \in \mathbb{Z}$ such that $m = q'n + a'$. As $0 \leq a' < n$, the uniqueness component of the division algorithm implies that $a' = a$. Therefore, there exists a unique $a \in \{0, \ldots, n-1\}$ such that $m \equiv a \pmod{n}$. □