Homework 4

MATH 301

Solution to graded problem

Exercise 1 (#46 in Section 3.5)**.** Prove or disprove: if H and K are subgroups of a group G, then $H \cup K$ is a subgroup of G.

Solution. The statement is false. To disprove a statement, we must show a counter example (that is, an example that satisfies the hypothesis but fails the conclusion). Let $G = \mathbb{Z}$, $H = 2\mathbb{Z}$, and $K = 3\mathbb{Z}$, so H and K are subgroups of $\mathbb{Z}$. Then, as $2 + 3 = 5$ is a neither a multiple of 2 nor 3, we know $5 \notin H \cup K$, which says that $H \cup K$ is not closed under the group operation and hence not a subgroup. □