

## Homework 7

MATH 301

Solution to graded problem

---

**Exercise 2** (Section 6.5 #18). If  $[G : H] = 2$ , prove that  $gH = Hg$ .

*Proof.* There are two cases: either  $g$  is in  $H$  or not. If  $g \in H$ , then  $gH = H$  and  $Hg = H$ ; hence,  $gH = Hg$ . Now, suppose that  $g \notin H$ . Then, as the left cosets of  $H$  form a partition of  $G$ , we have that  $G = H \cup gH$  and  $H \cap gH = \emptyset$ . Therefore,  $gH = G \setminus H$ . The same argument using right cosets implies that  $Hg = G \setminus H$  as well. Thus,  $gH = Hg$ .  $\square$