

Homework 1

MATH 301

Due Wednesday, February 8, 2022

Instructions. Read the [Homework Guide](#) to make sure you understand how to successfully complete the assignment. All claims must be sufficiently justified.

Exercise 1. Complete the following exercises from [Section 2.4](#) in the course textbook:

15, 16, 18, 20

Definition. Given two nonzero integers a and b , an integer c is a *common multiple* of a and b if $a \mid c$ and $b \mid c$. The *least common multiple* of a and b , denoted $\text{lcm}(a, b)$, is the smallest positive common multiple of a and b .

Exercise 2. Let a and b be nonzero integers.

- (1) Prove that the least common multiple of a and b exists.
- (2) Prove that if $k \in \mathbb{Z}$ is a common multiple of a and b , then $\text{lcm}(a, b)$ divides k . (Hint: divide k by m using the division algorithm.)
- (3) Prove that the product of $\text{lcm}(a, b)$ and $\text{gcd}(a, b)$ is equal to $|ab|$. (Hint: the product ab is divisible by $d = \text{gcd}(a, b)$. Let $m = |ab|/d$. Now, let k be a common multiple of a and b . Write d as a linear combination in a and b , and use this to express the fraction k/m as an integer.)
- (4) Prove that $\text{lcm}(a, b) = |ab|$ if and only if $\text{gcd}(a, b) = 1$.