

Homework 10

MATH 301

Solution to graded problem.

Exercise 2 (#11). Find five non-isomorphic groups of order 8.

Solution. Consider the five groups \mathbb{Z}_8 , $\mathbb{Z}_2 \times \mathbb{Z}_4$, $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$, D_4 , and Q_8 . The only cyclic group in the list is \mathbb{Z}_8 , so it is not isomorphic to any of the other four. Now $\mathbb{Z}_2 \times \mathbb{Z}_4$ and $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ are abelian, while D_4 and Q_8 are not abelian. So neither $\mathbb{Z}_2 \times \mathbb{Z}_4$ nor $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ is isomorphic to D_4 or Q_8 . Now, every non-identity element of $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ has order two, while $\mathbb{Z}_2 \times \mathbb{Z}_4$ has an element of order four; hence, they are not isomorphic. It is left to show that D_4 and Q_8 are not isomorphic. To see this, we have that Q_8 has only one element of order two, namely -1 , while D_4 has five elements of order two. We have established that no two groups in the list are isomorphic. \square