

## Homework 1

MATH 301

Solution to graded problem

---

**Exercise 2.** Let  $a$  and  $b$  be nonzero integers. Prove that if  $k \in \mathbb{Z}$  is a common multiple of  $a$  and  $b$ , then  $\text{lcm}(a, b)$  divides  $k$ .

*Proof.* Let  $m = \text{lcm}(a, b)$ . By the division algorithm, there exists  $q, r \in \mathbb{Z}$  such that  $k = qm + r$ , where  $0 \leq r < m$ . Write  $r = k - qm$ . Then, since  $a \mid k$  and  $a \mid m$ , we have that  $a \mid r$ . Similarly,  $b \mid r$ , and hence  $r$  is a common multiple of  $a$  and  $b$ . But if  $r > 0$ , then  $r$  would be a common multiple of  $a$  and  $b$  that is smaller than  $m$ , which is impossible. Therefore, we can conclude that  $r = 0$  and  $m \mid k$ .  $\square$