Homework 2

MATH 301

Due Wednesday, February 15, 2023

Instructions. *Read the Homework Guide to make sure you understand how to successfully complete the assignment. All claims must be sufficiently justified.*

Exercise 1. Complete the following exercises from Section 2.4 in the course textbook:

1, 5, 9, 31

Exercise 2. Complete the following exercises from Section 3.5 in the course textbook:

1 (a)–(d)

Exercise 3. Prove that mathematical induction holds. That is, prove that if $S \subset \mathbb{N}$ satisfies $1 \in S$ and $n+1 \in S$ whenever $n \in S$, then $S = \mathbb{N}$.

Exercise 4. Let $a, b, d \in \mathbb{Z}$ such that $d \mid a$ and $d \mid b$. Prove $d \mid (a+b)$.

(Note: the proof of this fact was embedded in a proof in class, and we have used this fact several times since. So it is good for you to have a proof once and for all as a separate statement.)

Exercise 5. Let p be a prime number and let $a_1, a_2, \ldots, a_k \in \mathbb{Z}$. Prove that if $p \mid a_1 a_2 \cdots a_k$, then there exists $i \in \{1, 2, \ldots, k\}$ such that $p \mid a_i$. (Hint: use induction and Euclid's lemma.)

Exercise 6. Let $n \in \mathbb{N}$. Prove that equivalence modulo n is an equivalence relation on $\mathbb{Z}$.

Exercise 7. Let $n \in \mathbb{N}$. Prove that given any $m \in \mathbb{Z}$, there exists a unique element $a \in \{0, 1, 2, \ldots, n-1\}$ such that $m = a \pmod{n}$. (Hint: use division algorithm.)