

## Homework 6

MATH 301

Due Wednesday, March 22, 2023.

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**Instructions.** Read the [Homework Guide](#) to make sure you understand how to successfully complete the assignment. All claims must be sufficiently justified.

**Exercise 1.** Let  $m, n \in \mathbb{N}$  be relatively prime. Prove that every homomorphism from  $\mathbb{Z}_m$  to  $\mathbb{Z}_n$  is trivial. (Recall: a homomorphism is *trivial* if it maps every element to the identity element.)

**Exercise 2.** Let  $h_1: \mathbb{Z}_4 \rightarrow \mathbb{Z}_6$  be defined by  $h_1([n]_4) = [3n]_6$  for  $n \in \mathbb{Z}$ , and let  $h_2: \mathbb{Z}_4 \rightarrow \mathbb{Z}_6$  be defined by  $h_2([n]_4) = [2n]_6$  for  $n \in \mathbb{Z}$ .

- (a) Prove that  $h_1$  is a *well-defined* function, that is, prove that, for any  $n, m \in \mathbb{Z}$ , if  $n \equiv m \pmod{4}$  then  $3n \equiv 3m \pmod{6}$ .
- (b) Prove that  $h_2$  is not a well-defined function, that is, find  $n, m \in \mathbb{Z}$  such that  $n \equiv m \pmod{4}$  but  $2n \not\equiv 2m \pmod{6}$ .
- (c) Explain why  $h_1$  is a homomorphism.
- (d) Find the kernel of  $h_1$ .

**Exercise 3.** Complete the following exercises from [Section 4.5](#) in the course textbook:

# 1(a,b,c), 2(a,e,f), 3(b,c,d,e), 4(a,b,c), 9, 11, 27, 30, 31, 39

**Exercise 4.** Complete the following exercises from [Section 5.4](#) in the course textbook:

# 1, 2(a,b,c,d), 5(ignore the first sentence, and just find each of the sets in a, b, and c, and decide whether they are subgroups or not)