

Homework 8

MATH 301

Solution to graded problem

Exercise 3 (#17). Suppose $[G : H] = 2$. If a and b are not in H , show that $ab \in H$.

Solution. As the index of H in G is 2, we know that H has two left cosets, namely H and bH . Since $a \notin H$ and H is closed under taking inverses, we must have that $a^{-1} \notin H$. In particular, $a^{-1} \in bH$, meaning there exists $h \in H$ such that $a^{-1} = bh$. Multiplying this equation on the left by a and on the right by h^{-1} , we have that $h^{-1} = ab$; hence, $ab \in H$. \square