

Homework 9

MATH 301

Solution to graded problem.

Exercise 6 (#39). Find $\text{Aut}(\mathbb{Z})$.

Solution. Let $\varphi \in \text{Aut}(\mathbb{Z})$ (i.e., $\varphi: \mathbb{Z} \rightarrow \mathbb{Z}$ is an isomorphism). Then, $\varphi(1)$ is a generator of \mathbb{Z} . There are only two generators of \mathbb{Z} , namely 1 and -1 . If $\varphi(1) = 1$, then φ is the identity automorphism, that is, $\varphi(n) = n$ for every $n \in \mathbb{Z}$. Otherwise, $\varphi(1) = -1$, and so $\varphi(n) = -n$ for every $n \in \mathbb{Z}$. Therefore, $\text{Aut}(\mathbb{Z})$ has exactly two elements, the identity automorphism and the automorphism given by $n \mapsto -n$ for $n \in \mathbb{Z}$. \square