## Homework 11 MATH 301/601

Due Wednesday, May 8, 2024

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**Instructions.** *Read the appropriate homework guide (Homework Guide for 301 or Homework Guide for 601) to make sure you understand how to successfully complete the assignment. All claims must be sufficiently justified.*

**Exercise 1.** Complete the following exercises from Section 10.4 in the course textbook:

# 1, 2, 3, 4, 5, 8, 9, ***11**

**Exercise 2.** Complete the following exercises from Section 11.4 in the course textbook:

# 9, 10, 13

***Exercise 3.** Let $\varphi\colon G_1 \to G_2$ be a homomorphism, let $H_2$ be a subgroup of $G_2$, and let $H_1 = \varphi^{-1}(H_2) = \{g \in G_1 : \varphi(g) \in H_2\}$.

(a) Prove that $H_1$ is a subgroup of $G_1$.

(b) Prove that if $H_2$ is normal in $G_2$, then $H_1$ is normal in $G_1$.

(Note: since the trivial subgroup is always normal, it follows that $\ker\varphi$ is a normal subgroup of $G_1$.)

***Exercise 4.** Let $G$ be a cyclic group, let $a$ be a generator of $G$, and let $\varphi, \psi\colon G \to H$ be homomorphisms. Prove that if $\varphi(a) = \psi(a)$, then $\varphi = \psi$. (This says that a homomorphism defined on a cyclic group is completely determined by its action on a generator of the group.)

**Exercise 5.** Let $\varphi\colon G \to H$ be a homomorphism. Prove that $\varphi$ is injective if and only if $\ker\varphi$ is trivial.

****Exercise 6.** The subgroup of a group $G$ generated by the set $\{xyx^{-1}y^{-1} : x, y \in G\}$ is called the *commutator subgroup of* $G$ and is denoted $G'$ (or $[G, G]$).

(a) Prove that $G'$ is normal in $G$.

(b) Prove that $G/G'$ is abelian.

(c) Let $N$ be a normal subgroup of $G$. Prove that $G/N$ is abelian if and only if $G' \subset N$.

(The group $G/G'$ is called the *abelianization* of $G$.)