Interpreting Proportional and Additive Differences in Mortality

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Abstract

Demographers and social scientists are interested in comparing well-being across groups and across time. Because mortality is a convenient measure of health, the literature on health inequalities frequently examines mortality but is mixed regarding whether proportional or additive differences are more appropriate indicators of inequality when examined over time. In this article, I argue that the dominant temporal pattern in mortality rates, one of proportional decline in mortality rates and linear increase in life expectancy, leads us to view proportional differences in mortality rates as the better indicator of health inequality.

1 Introduction

Researchers have devised many metrics of health, ranging from the most subjective, like self-reported health status, to the most objective, such as height, with many measures in between. Mortality is a good measure of population health because it is so clear-cut. As a result, mortality is often the statistic of choice for researchers examining social inequalities in health

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(Feldman et al., 1989; Duleep, 1989; Pappas et al., 1993; Preston and Elo, 1995; Schalick et al., 2000; Edwards, 2007).

At a point in time, relative well-being is easy to assess. A group facing a mortality rate of 2 percent is clearly worse off than one with a mortality rate of 1 percent because it is higher. The unresolved dilemma is how to measure and interpret differences in mortality rates when they are changing over time. Suppose the first group's mortality falls to 1 percent while the second group's falls to 0.4 percent. The additive difference in mortality between the groups has fallen, from 1 to 0.6 percentage point, but the proportional difference has increased, from a factor of 2 to 2.5. Has health inequality decreased or increased?

To be sure, this is basically a semantic point. The underlying mortality data are exactly the same regardless of whether we choose to measure differences in absolute or relative terms. But the measure that we choose, while perhaps a matter of aesthetics, will affect the conclusions that we draw about trends over time and their policy implications. We should promote the use of a single measure that is consistent with inequality as we typically perceive it, no small feat given the extensive array of inequality measures available (Allison, 1978; Cowell, 1995).

I argue that we should view health inequality as having increased in the simple example I presented above. Proportional differences in mortality rates better indicate health inequality because we usually refer to stable additive differences in life expectancy between discrete groups as indicating stable inequality. And as mathematical demography has revealed, stable additive differentials in life expectancy are roughly consistent with stable proportional differences in mortality rates. This is true both over time and in the cross section, a symmetry that further motivates the focus on proportional mortality differentials when assessing temporal trends.

In the sections that follow, I expand upon this argument in several brief stages. First, I review the evidence on temporal trends in mortality. Then I examine recent contributions in mathematical demography that explain these trends. Finally, I discuss the implications of these insights for research and policy.

2 Trends in mortality

Lee and Carter (1992) and Tuljapurkar et al. (2000) show that age-specific mortality rates have been declining exponentially across industrialized nations. Rates of decline have varied by country and by age, with faster rates at younger ages. But within each age group, proportional declines have been remarkably stable. Meanwhile, White (2002) revealed that life expectancy at birth has been increasing almost linearly in industrialized countries since 1955. Oeppen and Vaupel (2002) showed that the highest recorded female life expectancy has been following a linear trend for a much longer period, since 1830.

The most famous human mortality differential is the female advantage, which has persisted in the industrialized world for the past 200 years (Tabutin and Willems, 1998). The overall summary measure of this differential is the extra average life years enjoyed by females, or the sex gap, which is currently about 5 years in the U.S. when measured from birth. To be sure, researchers examine differences in mortality rates by age and sex to understand dynamics in the wedge (Pampel, 2002; Glei and Horiuchi, 2007), but the focal statistic remains the sex gap. We similarly measure racial and ethnic inequality in mortality as level differences in life expectancy, such as between whites and African Americans (Elo, 2001), between whom the additive gap has narrowed since 1940.

Given the temporal patterns in mortality, our perception of inequality in life expectancy as the additive difference is striking. In proportional terms, a fixed additive gap in life expectancy will shrink in importance as life expectancy increases. But viewed alongside one another, these trends in mortality across time and groups reveal a latent coherence. If all life expectancies were increasing linearly at the same rate over time, then additive differentials across groups at a point in time would remain fixed. Interpreting the additive gap between life expectancies as inequality is thus consistent with a prior that distributional stability requires groups to share equally in mortality decline, even if they do not share equally in life expectancy.

This seems like a reasonable prior, but how do fixed additive gaps in life expectancy compare with differences in mortality rates? The latter are more frequently the basis of comparison because they are simpler to construct. Mathematical demographers have explored the relationship between mortality rates, life expectancy, and changes in both, and their work provides further illumination.

3 Mortality rates and life expectancy

Life expectancy is a nonlinear function of age-specific mortality rates, and the relationship between differences in mortality rates and differences in life expectancies is complicated. But Pollard (1988) and Vaupel and Romo (2003) derive approximations to life expectancy that provide useful intuition.

Vaupel and Romo (2003) examine temporal change in life expectancy, but their results directly translate to cross-sectional differences as well. They show that the time derivative of life expectancy at birth at time t, $\dot{e}^o(0,t)$, can be expressed as

$$\dot{e}^o(0,t) \quad = \quad \bar{\rho}(t) \cdot e^\dagger(t) + \text{Cov}_f(\rho, e^o), \tag{1}$$

where $\bar{\rho}(t)$ is the average across ages of the percentage decline in age-specific mortality rates, which is typically a positive number; $e^\dagger(t)$ is the average number of life years lost due to death, which is a function of remaining life expectancy at each age; and where the covariance term captures the effects of more rapid mortality decline at particular ages. Although mortality rates generally decline at very different age-specific rates (Lee and Carter, 1992), the covariance term tends to be small. And while $e^\dagger(t)$ varies across space and time, to a first approximation it is fairly constant across modern populations, around 12 life years lost per death. Together, these observations suggest that

$$\dot{e}^o(0,t) \approx 12\ \bar{\rho}(t). \tag{2}$$

Equation (2) is a statement about the annual rate of change in life expectancy over time, which is roughly 12 times the annual average percentage decline in age-specific mortality rates. But we can reinterpret (2) as revealing the gradient of life expectancy across any variable x associated with mortality, rather than time:

$$\partial e^o(0,t)/\partial x \approx 12\ \bar{\rho}(x), \tag{3}$$

where $\bar{\rho}(x)$ is now the average across ages of the percentage change in mortality at x as x changes. This equation reveals that additive differences in life expectancy between groups defined by x is determined by proportional differences in their mortality rates, $\bar{\rho}(x)$. If the proportional differences in mortality remain the same over time — that is, if $\bar{\rho}(x)$ is independent of t — then so will the additive differences in life expectancy. Put another way, to maintain absolute differences in life span between groups, we desire no worsening in proportional inequality in mortality rates.

4 Discussion

Preston and Taubman (1994) point out that survivorship, or life, is the good that we seek to increase, not technically the inverse of mortality although the concepts are clearly related. But in assessing disparities in population health, it is often difficult to calculate an array of age-specific mortality rates, and thus life expectancy, for many subgroups with any degree of precision. Cell sizes are usually too small. As a result, researchers typically compare mortality rates rather than life expectancies for groups across a spectrum of values of a social indicator such as income or education.

Two commonly used measures are the slope index of inequality (SII) and the relative index of inequality (RII), both of which Wagstaff et al. (1991) review. The SII measures the absolute change in age-adjusted mortality between the bottom and the top of a distribution. An SII of -0.004, for example, would mean that the top enjoys a mortality rate lower by 0.004 than the bottom. The RII expresses that same gradient as a percentage of the average age-adjusted mortality rate, so that it represents the percentage change in mortality between the bottom and the top. If mortality in this example were 0.005 on average, then the RII would be $-0.004/0.005 = -0.8$, an 80 percent gradient.

The approximate relationship in equation (3) reveals a link between relative differences in mortality rates and additive differences in life expectancy. In this context, an increasing RII in mortality rates is associated with an increasing SII in life expectancies. The SII in mortality rates may be increasing, decreasing, or remaining the same depending on how fast mortality rates are declining overall. But the RII in mortality rates informs us about inequality in life expectancy, about which we have stronger priors.

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