

ECON 206  
MACROECONOMIC ANALYSIS

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Chapter # 5b

What we've learned so far

$$Y_t = \bar{A}K_t^{1/3}L_t^{2/3} \leftarrow$$

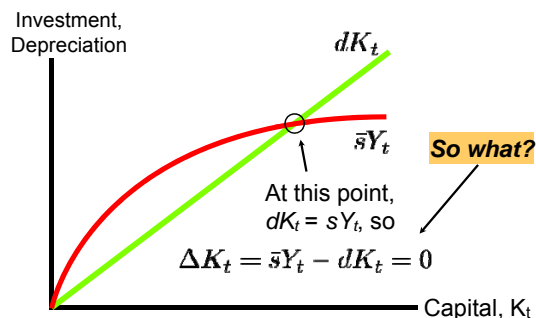
$$\Delta K_t = \bar{s}Y_t - dK_t \leftarrow$$

- The key equations of the Solow Model are these:
  - The production function
  - And the capital accumulation equation
- How do we solve this model?
  - We graph it, separating the two parts of the capital accumulation equation into two graph elements: **saving = investment**, and **depreciation**

The Solow Growth Model

Chapter 5 (2 of 2)

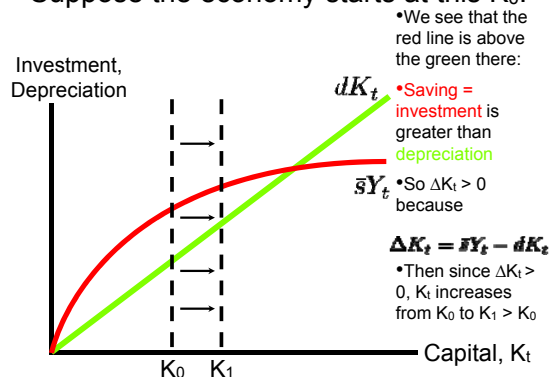
The Solow Diagram graphs these two pieces together, with  $K_t$  on the x-axis

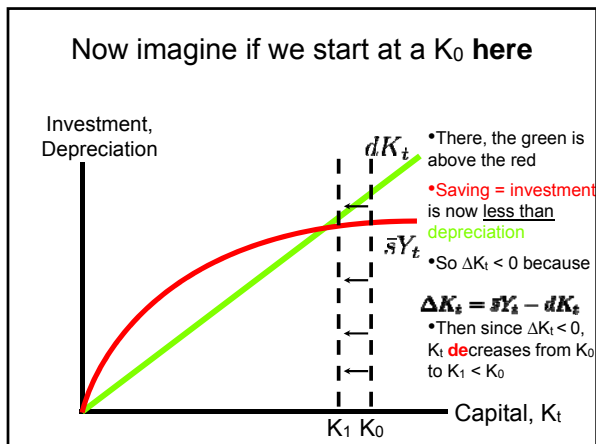


Our objectives today

- Finish up material from Chapter 5: The Solow Growth Model
- What does the Solow model tell us about how an economy grows over time?
- What is a steady state?
- What stuff affects the steady state? What happens when that stuff changes?
- What characterizes transition dynamics, or movement toward the steady state?
- What does the Solow model help us understand?

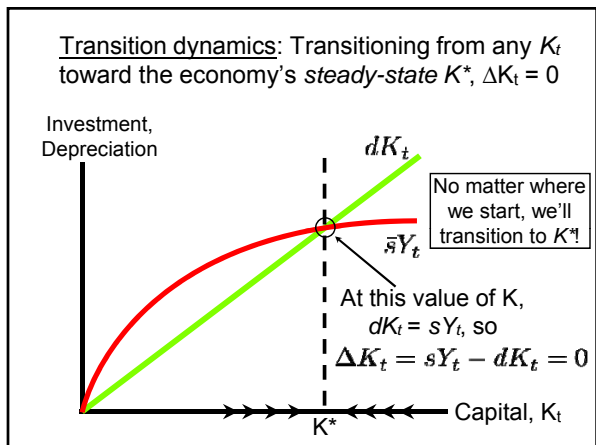
Suppose the economy starts at this  $K_0$ :





### What has the Solow Model revealed?

- Through the mechanism of saving and investing balanced against depreciation (machines breaking), capital accumulation moves the economy to a **steady state**, no matter how much or how little capital it started with
- Why? We reach a steady state because:
  - There are diminishing returns to capital: less  $Y_t$  per additional  $K_t$
  - That means new investment is also diminishing: less  $sY_t = I_t$
  - But depreciation is NOT diminishing; it's a constant share of  $K_t$
- After we reach the steady state, there is **no long-run growth** in  $Y_t$  (unless  $L$  or  $A$  increases): Saving and investing, a.k.a. capital accumulation, cannot produce long-run growth!



### What can we say about the steady state? What determines it?

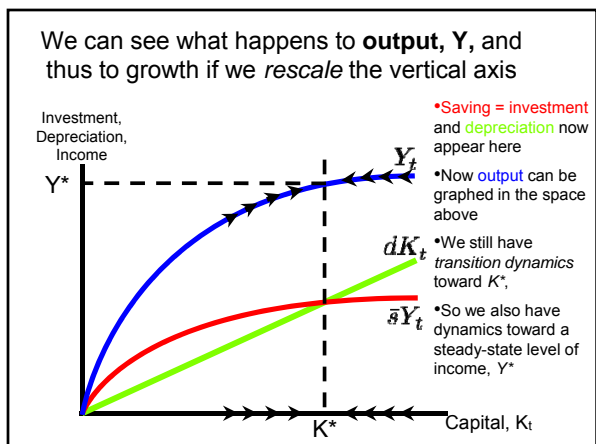
- We can solve *mathematically* for  $K^*$  and  $Y^*$  in the steady state, and doing so will help us understand the model better
- In the steady state, (and this is ALL in the book)
 
$$\Delta K_t = sY^* - dK^* = 0$$

$$sY^* = dK^*$$

$$s\bar{A}K^{*1/3}\bar{L}^{2/3} = dK^*$$

$$s\bar{A}\bar{L}^{2/3} = d\frac{K^*}{K^{*1/3}} = dK^{*2/3}$$

$$K^* = \left(\frac{s\bar{A}}{d}\right)^{3/2} \bar{L}$$



- If we know  $K^*$ , then we can find  $Y^*$  using the production function:  $K^* = \left(\frac{s\bar{A}}{d}\right)^{3/2} \bar{L}$
- This equation also tells us about income per capita,  $y$ , in the steady state:
 
$$Y_t = \bar{A}K_t^{1/3}\bar{L}^{2/3}$$

$$Y^* = \bar{A}\left(\frac{s\bar{A}}{d}\right)^{1/2}\bar{L}^{1/3}\bar{L}^{2/3}$$

$$Y^* = \left(\frac{s}{d}\right)^{1/2} \bar{A}^{3/2}\bar{L}$$

$$y^* \equiv \frac{Y^*}{\bar{L}} = \left(\frac{s}{d}\right)^{1/2} \bar{A}^{3/2}$$

$$y^* = \left(\frac{\bar{s}}{d}\right)^{1/2} \bar{A}^{3/2}$$

- Just like we did before with the simple model of production, we can use this formula to understand why some countries are so much richer
- As before, take the ratio of  $y^*$  for a rich country to  $y^*$  for a poor country, and assume the depreciation rate is the same across countries:
 
$$\frac{y_{rich}^*}{y_{poor}^*} = \left(\frac{\bar{A}_{rich}}{\bar{A}_{poor}}\right)^{3/2} \times \left(\frac{\bar{s}_{rich}}{\bar{s}_{poor}}\right)^{1/2}$$

$$45 = 18 \times 2.5$$
- Now we find that the factor of 45 that separates rich and poor country's income per capita is decomposable into ...
  - A  $10^{3/2} = 18$ -fold difference in this productivity ratio term,
  - And a  $(30/5)^{1/2} = 6^{1/2} = 2.5$ -fold difference in this investment rate ratio
- In the Solow Model, productivity accounts for  $18/20 = 90\%$  of differences!

Suppose the economy begins in steady state at  $K^*$  but the saving/investment rate rises from  $s$  to  $s' > s$

The graph plots Investment, Depreciation, and Income on the vertical axis against Capital,  $K_t$  on the horizontal axis. A blue curve represents the production function  $Y_t$ . A red curve represents the original investment function  $sY_t$ , and a green curve represents the new investment function  $s'Y_t$ . The original steady state is at  $K^*$  where  $sY_t = dK_t$ . The new steady state is at  $K^{**}$  where  $s'Y_t = dK_t$ . The transition from  $K^*$  to  $K^{**}$  is shown by a green arrow along the  $dK_t$  line.

- We start off at  $K^*$  and  $Y^*$  when the saving rate is  $s$
- A rise in saving from  $s$  to  $s' > s$  shifts the red curve upward
- The new steady state will be at  $K^{**}$  and  $Y^{**}$ , and we transition toward it over time

If productivity is still so important (and so far, unexplained), what good is the Solow model?

- It explains real-world capital-output ratios ( $K_t/Y_t$ ) well (Figure 5.3 in the book), so parts of it do match reality: specifically, its modeling of capital accumulation
- It provides new insights into growth:
  - Changes in the "fundamentals" — the exogenous variables in the model like the saving rate,  $s$ , and the depreciation rate,  $d$  — change the steady state and induce economic growth in the transition to the new steady state
  - This reveals the principle of transition dynamics: during the transition to the steady state, growth will be faster for the country that starts further from its steady state

What can we say about the rate of growth during the transition to the new steady state,  $K^{**}$  and  $Y^{**}$ ?

The graph is similar to the previous one, but it highlights the vertical distance between the investment curves  $sY_t$  and  $s'Y_t$  at various points during the transition. This distance represents  $\Delta K = s'Y_t - dK_t$ , which is the amount of capital accumulation. The graph shows that this distance is largest at  $K^*$  and shrinks as the economy moves toward  $K^{**}$ .

- After  $s$  has increased to  $s'$ , we are still at  $K^*$  temporarily
- $\Delta K = s'Y_t - dK_t$  is the vertical distance between red and green and is now very large
- So growth is rapid!
- As  $K$  increases, that distance shrinks, growth slackens

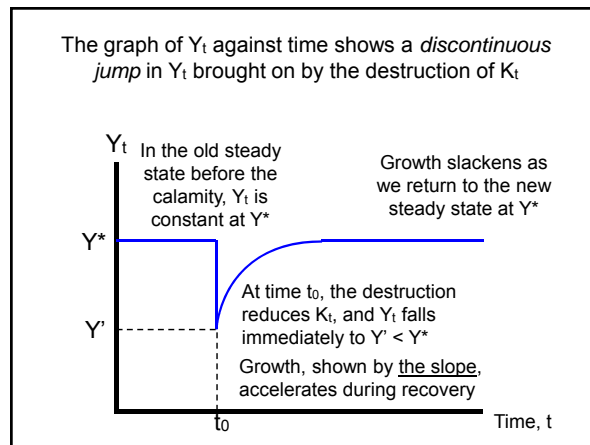
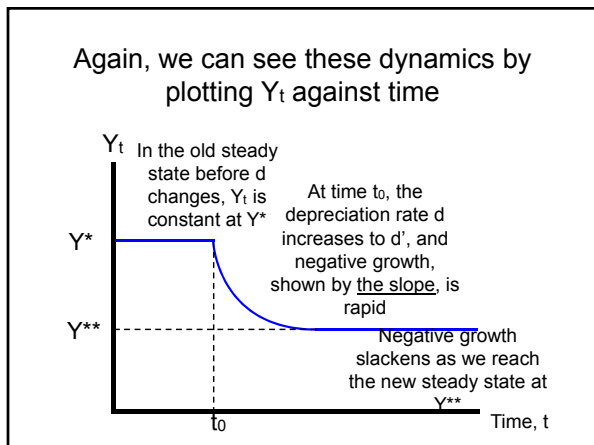
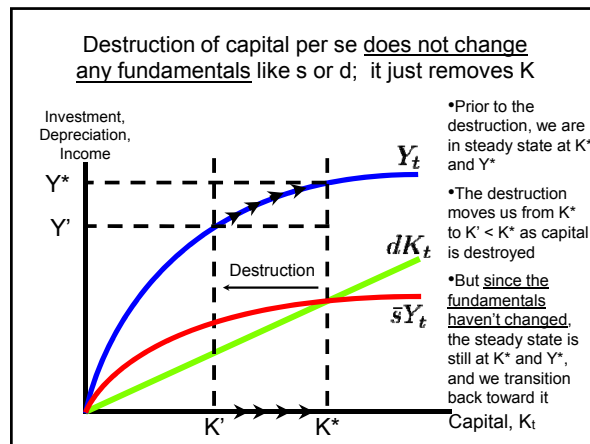
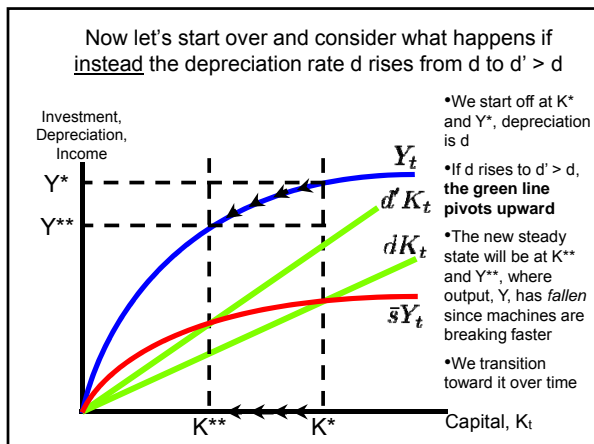
Putting the model to work: What happens if a fundamental, like  $s$  or  $d$ , changes?

- The model tells us about two things:
  - What the new steady state is (if there is one)
  - How the economy transitions to it
- We can see #1 using our mathematical formulas:
 
$$K^* = \left(\frac{\bar{s}\bar{A}}{d}\right)^{3/2} \bar{L} \quad Y^* = \left(\frac{\bar{s}}{d}\right)^{1/2} \bar{A}^{3/2} \bar{L}$$
- But we can only see #2 by graphing, and we can also see #1 that way, too
- You need to be able to do both, but graphing is more important

If we calculated and plotted  $Y_t$  against time using a ratio/log scale, we would see this:

The graph plots  $Y_t$  on the vertical axis against Time,  $t$  on the horizontal axis. The curve shows a constant level at  $Y^*$  until time  $t_0$ , where it jumps to a new constant level at  $Y^{**}$ . The slope of the curve is zero both before and after  $t_0$ , indicating that growth is zero in both steady states.

- In the old steady state before  $s$  changes,  $Y_t$  is constant at  $Y^*$
- Growth slackens as we reach the new steady state at  $Y^{**}$
- At time  $t_0$ , the saving rate  $s$  increases to  $s'$  and growth, shown by the slope, increases



Finally, this model helps us understand growth following the destruction of capital

- Why would we be interested in this application? What are examples of destruction of capital?
- Important (but tragic) world events involve capital destruction:
  - **Warfare** destroys productive capital — World War II resulted in the virtual obliteration of German and Japanese industrial capacity. And 9/11 resulted in the destruction of a lot of physical capital. (In both cases, the value of human lives lost surely dwarfed the value of lost capital.)
  - **Natural disasters** do too — One recent example in this country is Hurricane Katrina in 2005, which destroyed capital and killed people in New Orleans and along the Gulf Coast
- How do we think about the destruction of capital in the Solow Model? What does it imply about initial effects and recovery?

What have we discovered about the relative rate of growth?

- Growth is faster when the economy is further below its steady-state
- (Negative growth, or shrinking, is faster when the economy is further above its steady state — but such cases are rare)
- Let's consider the first case: Suppose country A is only 10% below its steady state, while country B is 50% below. Then country B will **grow faster** than country A, even though both are growing, because B is further away from its steady state
- We call this "the principle of transition dynamics"

### Why is the principle of transition dynamics useful?

- It describes the postwar experience of industrialized economies pretty well:
  - Japan and Germany grew extremely fast as reconstruction commenced following the destruction of their capital stocks, much faster than even the U.S.
  - But as these countries approached their steady states, their growth rates fell
- But unfortunately, we cannot explain (the lack of) growth in most developing countries using this principle — they seem to be *already trapped* in their low steady states rather than growing through transitioning to steady state

### But what *does* the Solow Model reveal about growth in developing countries?

- Although capital accumulation *cannot account* for the massive differences in income per capita between rich and poor countries,
- It *can* help us understand why we see some developing countries growing more rapidly than others
- How? Some countries, like the Republic of Korea, have experienced large increases in the saving/investment rate during the postwar period (*Why? Demography?*)
- As we have seen, if  $s$  is higher, steady-state income per capita is higher — and transitioning toward there would entail relatively faster growth

### Next: What *does* explain sustained economic growth?

- So far, we have models that tell us nothing about what really matters for sustaining growth over the long run
- Then what does matter? **Productivity** and its determinants