ECON 206
MACROECONOMIC ANALYSIS

Roumen Vesselinov

Chapter # 4a

Cross-country income differences are large, and we want to understand them

- Today, residents of the U.S. are:
- 4 times richer than residents of Brazil
- 9 times richer than residents of China
- 50 times richer than those in the poorest countries
- Why?
- We could tell many stories, but how do we assess which are more pertinent?
- To seek an explanation, we need a **model**

A Model of Production

Chapter 4 (1 of 2)

What will this model look like?

- A set of several mathematical equations
- These equations have graphical expressions too, but the math is meant to be simple while providing insights and numeric predictions
- For example: the model will assert that given the inputs, GDP per capita should be $20,000
- The model will be **highly simplified** and a clear abstraction from reality. (But that's perfectly OK)

Our objectives today

- Set up and solve a macroeconomic model that we will use to understand differences in GDP per person across countries
- Think about "returns to scale" and "diminishing marginal productivity" of inputs like capital
- *Next time*, part 2 of Chapter 4:
- Use the model to understand differences in GDP per capita across countries
- Gauge the importance of physical capital versus total factor productivity in explaining those differences

The setup of the production model

- Suppose the economy produces only one good, which happens to be **ice cream**
- There are a certain number of workers available to produce ice cream, $\bar{L}$ (which is exogenously fixed)
- There are also a number of ice cream machines, $\bar{K}$
- Workers use ice cream machines to make ice cream according to a **production function**, $F(K, L)$

A production function

- This function takes as **inputs** the number of workers, L, and the number of machines, K
- And returns as **output** the amount of ice cream, that is produced by those workers using those machines

$$Y = F(K, L) = \bar{A}K^{1/3}L^{2/3}$$

- $\bar{A} > 0$ is a positive constant term, which we will call **productivity** — because when it is higher, output is higher even with the same K and L

How does this economy allocate its resources given the production function?

- If markets are perfectly competitive, then firms in the economy will *maximize their profits*
- What characterizes profit maximization?
- From intermediate microeconomics:
- Firms will employ labor, L, and capital, K, up to the point at which their prices equal their marginal products
- A key element: *diminishing marginal productivity* of K and L

$$Y = F(K, L) = \bar{A}K^{1/3}L^{2/3}$$

- It is a *Cobb-Douglas* production function, and it is the most commonly used model in all of economics
- Why? It has very convenient **properties**
- The first key property is constant returns to scale:
- If you increase all inputs by some proportion, you will increase output by the very same proportion
- In other words, if K doubles and L doubles, then Y doubles too

What is diminishing marginal productivity of a factor like capital, K, or labor, L?

- "Marginal productivity" means the addition to output, Y, obtained from an additional unit of the factor K or L
- "Diminishing" means it falls as K or L rises

Why does this matter? Why do we care?

- It is realistic, as we discuss on the next slide
- It plays a key role in understanding international differences in output per person

The math of constant returns

$$\begin{aligned} F(2K, 2L) &= \bar{A}(2K)^{1/3}(2L)^{2/3} \\ &= 2^{1/3}2^{2/3}\bar{A}K^{1/3}L^{2/3} \\ &= 2^{1/3+2/3}\bar{A}K^{1/3}L^{2/3} \\ &= 2F(K, L) \end{aligned}$$

- Why are constant returns desirable in a model?
- If you *duplicate* your plant and workers, you would expect to *duplicate* output

Why might marginal productivity diminish?

- Suppose our ice cream shop has **five** workers and only **one** ice cream machine
- Workers probably bump into one another or are idle
- One more machine probably increases output *by a lot*
- Suppose we added a machine and now have **two**
- Workers don't bump into each other very much
- But one more machine *now* probably increases output only some
- What if we have **five** machines and **five** workers
- One more machine *might sit idle, totally unused!*
- Output wouldn't increase at all

What is marginal productivity in this model?

- The marginal product of capital is given by the following mathematical equation (which you do not need to memorize):

$$MPK = \frac{1}{3}\bar{A}\left(\frac{L}{K}\right)^{2/3} = \frac{1}{3} \times \frac{Y}{K}$$

- You can see that when you hold the number of workers, L, constant, then MPK declines when K increases
- It turns out that mathematically, MPK also equals ...
- This is another special property of the Cobb-Douglas production function that will be useful later

What have we learned?

1. How to set up and solve a model
   - This model is simple: the supplies of K and L are constant at $\bar{K}$ and $\bar{L}$, and demand only determines their prices, r and w
   - But we will build on this model in future chapters
2. Output, Y, is determined by capital, K, and labor, L
   - If an economy has more machines or more people, it should have more output
   - We now know more about output per person
3. We understand wages, w, and interest rates, r ...

Everything we have said about capital applies to labor as well

- Suppose we increased the number of workers while keeping the number of ice cream machines constant
- Each additional worker would start bumping into the other workers more and more, since there wouldn't be enough machines to go around
- The marginal product of *labor* is very similar to the MPK:

$$MPL = \frac{2}{3}\bar{A}\left(\frac{K}{L}\right)^{1/3} = \frac{2}{3} \times \frac{Y}{L}$$

Properties of the wage, w, and the interest rate, r

- The wage is proportional to output per worker
- Likewise, the interest rate is proportional to output per unit of capital
- Rearranging MPK = r and MPL = w, we can see that:

$$\frac{wL}{Y} = \frac{2}{3} \qquad \frac{rK}{Y} = \frac{1}{3}$$

- In words, two-thirds of output is paid to labor while one-third is paid to capital
  - We saw this fact graphically in Class #2 in Figure 2.3
  - It explains why we chose the exponents that we did

Model's solution as a set of equations

- Firms employ capital and labor until their prices, the interest rate **r** for capital, K, and the wage rate **w** for labor, L, equal their marginal products:

$$MPK = \frac{1}{3} \times \frac{Y}{K} = r \quad MPL = \frac{2}{3} \times \frac{Y}{L} = w$$

- We also know the production function:

$$Y = F(K, L) = \bar{A}K^{1/3}L^{2/3}$$

- And we know the amounts of capital and labor:

$$K = \bar{K} \quad L = \bar{L}$$

- Another implication of the model's math is that:

$$wL + rK = Y$$

- In words, the sum of payments to labor and capital is equal to total production in the economy
- This also states that total income — the sum of income paid to labor plus income paid to capital — is equal to total production, as we learned earlier when studying GDP
- Income and production are also equal to spending in this toy-model economy: all ice cream is consumed (there is no investment, no government, no imports, and no exports)

Next, we will use this model to understand why some countries are so much richer than others

- What does the model say about per capita GDP?
- Let's use lowercases to show variables per capita. Then our model reveals that

$$y \equiv \frac{Y}{L} = \frac{\bar{A}\bar{K}^{1/3}\bar{L}^{2/3}}{\bar{L}} = \frac{\bar{A}\bar{K}^{1/3}}{\bar{L}^{1/3}} \equiv \bar{A}\bar{k}^{1/3}$$

- Output per worker, y, is a function of productivity, A, and capital per worker, k = K/L
- (But there are diminishing returns to capital per worker!)
- Our model says countries are **richer** if A or k is higher

Next class:

- We use this model that we worked so hard to set up and understand:
- We fit it to data on GDP per capita across countries of the world
- We assess the fit of the model, and learn a lot in the process of doing so