ECON 206
MACROECONOMIC ANALYSIS

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Chapter # 5a

A recap of last class:

- Our simple production model told us that differences in income per person (y) cannot be adequately explained by differences in capital per person (k)
- We defined **total factor productivity** (A) as the residual in our formula for y:

$$y = \bar{A}\bar{k}^{1/3} \longrightarrow \bar{A} = \frac{y}{\bar{k}^{1/3}}$$

- We found that **TFP** accounts for two-thirds of the differences in income per capita (y), while capital per worker (k) only accounts for one-third

The Solow Growth Model

Chapter 5 (1 of 2)

Since productivity matters so much, **why** are some countries more productive than others?

- Total factor productivity captures the effects of *all elements other than physical capital per worker:*
  - Natural resources like oil and diamonds
  - Human capital: education and skills
  - Technology: knowledge, techniques
  - Institutions: policies, laws, regulations, organizations
- Natural resources are important for a few countries, but human capital, technology, and institutions are more broadly important elements that we now discuss

Our objectives today

- Chapter 5 (today and next class):
- How capital accumulates over time and contributes to economic growth
- The role of the diminishing marginal product of capital in explaining differences in growth rates across countries
- How a country's rate of growth depends on how far away it is from its *steady state* (we will define what this means)
- The limitations of capital accumulation and how it cannot explain a significant part of economic growth and cross-country income differences

Human capital

- As you are all well aware, education produces a stock of knowledge and skills that raise your productivity and wages, thus:
- This is clearly a factor of production in addition to just the number of warm bodies
- Accounting for differences in average years of education between countries **does reduce** unexplained TFP differences, but probably only cuts TFP's importance in half, which still leaves a lot unexplained

Technology

- Industrialized countries produce very different types of goods using very different techniques than developing countries
- So it could be the case that differences in total factor productivity reflect fundamental differences in technology
- We will discuss this in later in Chapter 6

Let's explore the intuition behind the model's approach with an example

- What is the point of this example? We want to understand where capital comes from and how we should think about capital. First, let's be clear: *what is capital?*
- *Capital* is a catch-all term referring to physical factors of production like machinery, factories, computers, etc.
- What is special about capital as opposed to other goods?
- *Once you have capital, it is durable;* it "sticks around" rather than vanishing immediately
  - Think of a car compared to a bag of chips: You drive the car once, but it doesn't explode or anything. But if you eat the chips, *they're gone*
  - Capital goods are like cars: you use them once, then maybe there's some wear-and-tear, but then you can use them again!

Institutions

- Some of the best evidence we have about the importance of institutions comes from the aftermath of wars and other "accidents of history"
  - North and South Korea, East and West Germany
  - These countries were essentially dealt different socio-political systems, but they had similar background circumstances
  - After a very brief time, **huge differences** in GDP per capita
- What's going ? What might be important institutions?
  - Property rights, the rule of law, contract enforcement, separation of power and checks & balances *all matter for growth*
  - Quantifying these effects is at the forefront of research!

The example: Imagine the economy consists of a **farm** that produces **corn**

- Each harvest, the family on the farm reaps their corn
- Now they're hungry, and they want to eat some of the corn!
- But if they consumed all their corn, there wouldn't be any **seed corn** — corn that you must *plant* to produce new corn at the next harvest
- So the family eats three-quarters of their corn and saves one-quarter, placing that 1/4 in a silo for planting later
- What represents what?
  - All corn harvested is output = income (Y)
  - The eaten corn is consumption (C)
  - ★ The seed corn in the silo is saving = investment (I) [= capital (K)] [*Reality is more complicated because not all capital is used up each year like all seed corn is*, so usually we see that K > I]

The Solow Growth Model

- Why are we studying this?
- Robert Solow won the Nobel Prize in 1987 because of this theory — one of the most widely used in macro
- The model is extremely useful for issues in development economics and population economics — or what the World Bank and UN do on a daily basis
- The Solow Model builds on the production model of Chapter 4, this time explaining why we have the capital stock that we do, rather than just taking it as given (so we *endogenize* capital)
- But the model isn't perfect — for example, it does not help us understand **TFP**, which we know we need to do (in due time)

Now let's formalize this in a model

- Just like before, the economy produces output Y using capital K and labor L and a production function Y = F(K,L):

$$Y_t = F(K_t, L_t) = \bar{A}K_t^{1/3}L_t^{2/3}$$

- Like corn, output Y is either consumed (eaten) or invested (planted):

$$Y_t = C_t + I_t$$

- What does investment (I_t) do? It produces **new capital** that increases the capital stock, K_t. How?

$$K_{t+1} = K_t + I_t - d \times K_t$$

- The amount of capital (machines, equipment, plants) that we have *next period* (t+1)
- is equal to the amount of capital we had this period,
- plus **new investment**, which produces new capital,
- *minus* "depreciation" — the share of capital that breaks during this period because we used it. We call this share "*d*," so *d* x K_t units will break

What else is in the model?

- Just like in the last model, we will assume that labor, L, is just given to us and exogenous

$$L_t = \bar{L}$$

- And the last piece: what determines the level of investment (I_t)?
- Like the farmers with the corn, we will assume that our economy saves some fixed fraction "s" of its income, which becomes investment

$$I_t = \bar{s}Y_t$$

$$K_{t+1} = K_t + I_t - d \times K_t$$

- Notice that we can rewrite this equation to find a new equation for the *change in* K_t (and we'll remove the × and just write dK_t):

$$\Delta K_t = I_t - dK_t$$

- Why do this? It will become handy later

Let's list the equations in the model

$$Y_t = \bar{A}K_t^{1/3}L_t^{2/3}$$
$$L_t = \bar{L}$$
$$\Delta K_t = I_t - dK_t \quad \leftarrow$$
$$I_t = \bar{s}Y_t \quad \leftarrow$$
$$\Delta K_t = \bar{s}Y_t - dK_t$$

- Notice that I_t appears in two equations, which we can combine
- But in general, you can't find "a solution" for this model — not in one single equation, anyway

$$\Delta K_t = I_t - dK_t$$

- How does this work? Let's use a spreadsheet
- Suppose we start with K_0 = 1000, and each period, investment I_t = 200 and depreciation *d* = 0.1 = 10%

time	K_t	I_t	dK_t	ΔK_t	K_{t+1}
0	1000	200	100	100	1100
1	1100	200	110	90	1190
2	1190	200	119	81	1271

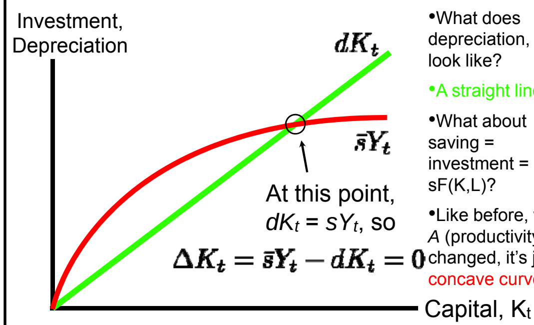
How do we proceed?

- We will **solve** the Solow model **graphically**

$$\Delta K_t = \boxed{\bar{s}Y_t} - \boxed{dK_t}$$

- We will split the right-hand side of this equation into its two pieces:
  - Saving = Investment
  - Depreciation
- Our strategy: graph *both* of these two pieces together, then their intersection means ΔK_t = 0

The Solow Diagram graphs these two pieces together, with K_t on the x-axis



- What does depreciation, dK_t, look like?
- A straight line!
- What about saving = investment = sY_t = sF(K,L)?
- Like before, when *A* (productivity) changed, it's just a concave curve

Next time: Using the Solow Model

- What does the model tell us about how an economy grows over time?
- What is a *steady state*?
- What stuff affects the steady state? What happens when that stuff changes?
- What characterizes *transition dynamics*, or movement toward the steady state?