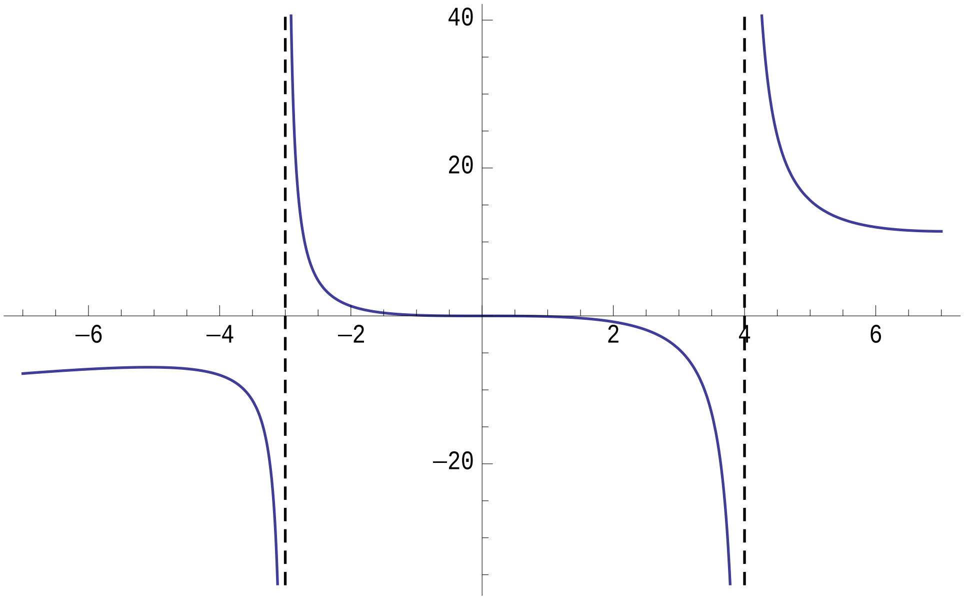
MATH 141, Calculus/Differentiation, Fall 2024
Exam I by Scott Wilson

Name: ____

Problem	Max points	Grade
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Instructions: **Read each problem carefully**. Show all of your work, in order to receive full or partial credit. If you need more space, you can use the back of the pages. In this case, make a clear reference to the continuation of your work. Graphing calculators are permitted on this exam.
You may use any results that were proven in class or in the homework. In this case, clearly state what result you are using and how it applies.

(1) Use the graph of the function f, given below, to answer each part.



(a) What is the domain of this function? All real numbers except -3 and 4.

(b) Is f continuous at 3? Yes.

(c) Is f continuous on the interval $[2, 6]$? No, f is not defined at 4.

(d) What is $f(0)$? $f(0) = 0$.

(e) What is $\lim_{x \to 4^-} f(x)$? $-\infty$.

(2) Answer each part.

(a) Let f be any function which is defined near some number a. According to the definition, what conditions must be true for f to be continuous at a ?
[I am asking you to *give the definition of "f is continuous at a"*.]

The function f is continuous at a if all of the following conditions hold:
(i) $f(a)$ is defined,
(ii) $\lim_{x\to a} f(x)$ exists, and
(iii) $\lim_{x\to a} f(x) = f(a)$
You could have written just the last condition, which presumes that both sides of the equation make sense (i.e. the first two conditions hold) but I have chosen to be very clear here.

(b) Consider the function

$$f(x) = \begin{cases} x^2 + 3x + 1 & \text{if } x \neq 2 \\ A & \text{if } x = 2 \end{cases}$$

defined on the interval $[0, 4]$. Find all value(s) of A for which the above function is continuous on the entire interval $[0, 4]$. Justify your answer.

The function $x^2 + 3x + 1$ is continuous on $[0, 4]$ since it is a polynomial. So, in order for f to be continuous we must have $\lim_{x\to 2} f(x) = f(2) = A$. Since

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} x^2 + 3x + 1 = 2^2 + 3 * 2 + 1 = 11$$

then we must have $A = 11$.

(3) Use the Intermediate Value Theorem to show that there is a real number which is a solution to the equation

$$2x^5 + x^3 = 7$$

Carefully explain your reasoning, including an explanation of how the theorem applies.

First, the function $f(x) = 2x^5 + x^3$ is a polynomial so it is continuous for all real numbers. By the statement of the IVT, it suffices to find two values for x , say a and b, for which $f(a) < 7$ and $f(b) > 7$. We can choose $a = 0$ since $f(0) = 0 < 7$, and we can choose $b = 2$ since $f(2) = 72 > 7$. So, by the intermediate value theorem, there is some number c between 0 and 2 for which $2c^5 + c^3 = 7$.

Instead you could consider the function $g(x) = 2x^5 + x^3 - 7$, and use that $g(0) = -7 < 0$,and $g(2) = 65 > 0$, so that $g(c) = 0$ for some c between 0 and 2, and then $2c^5 + c^3 - 7 = 0$, so that $2c^5 + c^3 = 7$.

(4) Calculate the following limits. Your answer should be a real number, or ∞, or $-\infty$, or if none of these hold, write DNE and explain.

(a)

$$\lim_{x\to 3} \frac{x^2+x-12}{x-3}$$

$$\lim_{x\to 3} \frac{x^2+x-12}{x-3} = \lim_{x\to 3} \frac{(x+4)(x-3)}{x-3} = \lim_{x\to 3} x+4 = 7$$

The first equality follows from factoring, the second equality follows since for $x \neq 3$ we have $\frac{(x+4)(x-3)}{x-3} = x+4$, so the limits as $x \to 3$ are equal, and the last equality follows from continuity of $x+4$ at 3.

(b)

$$\lim_{x\to 4^-} \frac{x-4}{|x-4|}$$

For all numbers x close to 4 and less than four we have $\frac{x-4}{|x-4|} = -1$ so

$$\lim_{x\to 4^-} \frac{x-4}{|x-4|} = -1$$

(5) Calculate the following limits. Your answer should be a number, or ∞, or $-\infty$, or if none of these hold, write DNE and explain.

(a)

$$\lim_{x\to\infty} \sqrt{9x^2+x} - 3x$$

$$\begin{aligned}
\lim_{x\to\infty} \sqrt{9x^2+x} - 3x &= \lim_{x\to\infty} \left(\sqrt{9x^2+x} - 3x\right) \frac{\sqrt{9x^2+x}+3x}{\sqrt{9x^2+x}+3x} \\
&= \lim_{x\to\infty} \frac{x}{\sqrt{9x^2+x}+3x} \\
&= \lim_{x\to\infty} \frac{1}{\sqrt{9+1/x}+3} \\
&= \frac{1}{\sqrt{9+\lim_{x\to\infty} 1/x}+3} \\
&= \frac{1}{\sqrt{9+0}+3} \\
&= \frac{1}{6}
\end{aligned}$$

(b)

$$\lim_{x\to\infty} \frac{2x^3+5x+1}{x^3-x^2}$$

$$\lim_{x\to\infty} \frac{2x^3+5x+1}{x^3-x^2} = \lim_{x\to\infty} \frac{\frac{2x^3}{x^3}+\frac{5x}{x^3}+\frac{1}{x^3}}{\frac{x^3}{x^3}-\frac{x^2}{x^3}} = \lim_{x\to\infty} \frac{2+\frac{5}{x^2}+\frac{1}{x^3}}{1-\frac{1}{x}} = 2$$

since $1/x^n \to 0$ as $x \to \infty$.