Math 120 **Name:** ____________________
Fall 2015
Exam #1
10/15/2015
Time Limit: 75 Minutes

You may *not* use your books or notes on this exam. You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Grade Table (for teacher use only)

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	0	
Total:	80	

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1. (20 points) Construct a truth table for the following compound proposition:

$$(p \wedge q) \leftrightarrow (p \to \neg q)$$

p	q	$(p \wedge q)$	$(p \to \neg q)$	$(p \wedge q) \leftrightarrow (p \to \neg q)$
T	T	T	F	F
T	F	F	T	F
F	T	F	T	F
F	F	F	T	F

2. (20 points) Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output $(p \wedge \neg r) \vee (\neg q \wedge r)$ from input bits p, q, and r.

   I'm going to do this one in class, you can draw this solution in a few different ways.

3. (20 points) (a) (5 points) When are two propositional statements *logically equivalent*? I took a few different answers for this question. I was willing to take the textbook definition, and I was willing to take something along the lines of: *two propositions are equivalent if they have the same truth values (as statements) when their propositional variables have the same truth values.*

   (b) (15 points) Show that

$$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$$

It is easiest to do this using a truth table.

p	q	r	$p \to q$	$p \to r$	$q \wedge r$	$p \to (q \wedge r)$	$p \to q) \wedge (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	F	F	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

4. (20 points) (a) (5 points) Translate this nested quantifications into an English statement that expresses a mathematical fact. The domain in consists of all real numbers.

$$\exists x \forall y (xy = 1)$$

There is a real number x such that for every real number y, $xy = 1$. This is of course, not true.

(b) (5 points) How is the nested quantifyer in (a) different from the following nested quantifyer?

$$\forall x \exists y (xy = 1)$$

This one says that, for every real number x there exists a real number y such that $xy = 1$. This is true.

(c) (5 points) Express the negation of the following statement such that no negation sign preceeds a quantifyer.

$$\exists z \forall y \forall x T(x, y, z)$$

Using rules establed for the negation of quantifyers,

$$\forall z \exists y \exists y \neg T(x, y, z)$$

(d) (5 points) Consider the following statement: "If n is a real number such that $n > 1$, then $n^2 > 1$". I have a proof for this statement, and here it is: *Suppose that* $n^2 > 1$. *Then* $n > 1$. Is this reasoning correct? Why or why not?

Of course not, I did the following:

$$P \rightarrow Q$$

$$Q$$

$$\therefore P$$

This is not a valid argument.

5. (20 points) (a) (10 points) Prove directly that the product of two rational numbers is a rational number.
   Let x and y be rational numbers, ie

$$x = \frac{a}{b}, y = \frac{c}{d}$$

   Where $a, b, c, d \in \mathbb{Z}$ and $b, d \neq 0$. Then,

$$xy = \frac{ac}{bd}$$

   Which must be rational, as ac and bd are integers, and bd must not be zero, as b and d were not zero.

   (b) (10 points) Prove that if x is an irrational number (recall: an irrational number can *not* be put in the form $\frac{a}{b}$ where a, b are integers and $b \neq 0$), then $\frac{1}{x}$ is an irrational number.
   Suppose that x is irrational, and that $\frac{1}{x}$ is a rational number (I am using the 'proof by contradiction method' here). In this case,

$$\frac{1}{x} = \frac{a}{b}$$

   Where a, b are integers and $b \neq 0$. Note that $a \neq 0$, as x is assumed to be irrational. Now,

$$x = \frac{1}{\frac{1}{x}} = \frac{1}{\frac{a}{b}} = \frac{b}{a}$$

   Which is a rational number. Thus we have a contradiction, $\frac{1}{x}$ must be irrational.